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**Etienne Augé,
Jacques Dumarchez
and
Jean Trân Thanh Vân**

Atom interferometry and the Einstein equivalence principle

Peter WOLF¹, Luc BLANCHET², Christian J. BORDÉ^{1,3},
Serge REYNAUD⁴, Christophe SALOMON⁵, Claude COHEN-TANNOUDJI⁵

¹*LNE-SYRTE, Observatoire de Paris, CNRS, UPMC, France*

²*GRECO, Institut d'Astrophysique de Paris, CNRS, UPMC, France*

³*Laboratoire de Physique des Lasers, Université Paris 13, CNRS, France*

⁴*Laboratoire Kastler Brossel, CNRS, ENS, UPMC, France*

⁵*Laboratoire Kastler Brossel et Collège de France, CNRS, ENS, UPMC, France*

The computation of the phase shift in a symmetric atom interferometer in the presence of a gravitational field is reviewed. The difference of action-phase integrals between the two paths of the interferometer is zero for any Lagrangian which is at most quadratic in position and velocity. We emphasize that in a large class of theories of gravity the atom interferometer permits a test of the weak version of the equivalence principle (or universality of free fall) by comparing the acceleration of atoms with that of ordinary bodies, but is insensitive to that aspect of the equivalence principle known as the gravitational redshift or universality of clock rates.

The Einstein equivalence principle is at the basis of our understanding of modern theories of gravity¹. The weak version of the equivalence principle, also called universality of free fall (UFF), has been verified with high precision using torsion balances^{2,3,4} and the Lunar laser ranging⁵. Atom interferometry experiments have also yielded an important test of the UFF at the level 7×10^{-9} , by comparing the acceleration of atoms with that of ordinary bodies^{6,12,7}.

The gravitational redshift of universality of clock rates (UCR) is the least tested aspect of the equivalence principle. It is currently known with 10^{-4} accuracy⁸ and should be tested at the level 10^{-6} in future space experiments with clocks⁹. In this contribution we report arguments^{10,11} showing that a recent claim¹² that atom interferometry experiments have actually already tested the UCR at the level 7×10^{-9} (thereby improving the validity of the redshift by several orders of magnitude, even with respect to future space experiments⁹), is fundamentally incorrect. More recently, our arguments have received support from several independent analyses^{13,14}.

In conventional clock experiments^{8,9}, the measurement of the gravitational redshift uses two clocks A and B located at different heights in a gravitational field, and operating at the frequency ω of an atomic transition. The two measured frequencies ω_A and ω_B are continuously compared through the exchange of electromagnetic signals. These measurements rely on atomic spectroscopy (Cæsium clocks, hydrogen masers, optical clocks, etc.) or nuclear spectroscopy (like in the Pound-Rebka experiment¹⁵). The two clocks are put in devices (experimental setups, rockets, satellites, etc.) that are classical and whose trajectories can be measured by radio or laser ranging. The atomic transition of A and B used as a frequency standard is described *quantum mechanically* but the motion of A and B in space can be described *classically*. The motion of the two clocks can thus be precisely measured and the contribution of the special relativistic term (i.e. the Doppler effect) can be evaluated and subtracted from the total frequency

shift to get a test of the gravitational redshift.

In the proposal¹² the atoms in an atom interferometer are considered as “clocks” ticking at the Compton frequency $\omega_C = mc^2/\hbar$ associated with their rest mass, and propagating along two “classical” arms of the interferometer. However we dispute this interpretation^{10,11}: (i) An atom is not a “Compton clock”, since it does not deliver a physical signal at the Compton frequency^{11,13}. (ii) In an atom interferometer, we are using an interference between two possible paths followed by the *same* atom, which is described quantum mechanically. Contrary to clock experiments the motion of atoms is not monitored. It is deduced from the theory by using the same evolution equations which allow one to evaluate the phase shift. Thus the two classical paths in the interferometer cannot be determined by a measurement. In an interferometer, where a single atom can propagate along two different paths, trying to measure the path which is followed by the atom destroys the interference signal (wave-particle complementarity). (iii) Using the theory in a consistent manner, the contribution to the phase shift which depends on the mass of the atom (and therefore on its Compton frequency), and includes a contribution from the gravitational redshift, is in fact exactly zero^{17,21,18}.

The Cæsium (or some alkali) atoms are optically cooled and launched in a vertical fountain geometry. They are prepared in a hyperfine ground state g . A sequence of vertical laser pulses resonant with a $g \rightarrow g'$ hyperfine transition is applied to the atoms during their ballistic (i.e. free fall) flight. In the actual experiments the atoms undergo a two-photon Raman transition where the two Raman laser beams are counter-propagating. This results in a recoil velocity of the atoms, with the effective wave vector k transferred to the atoms being the sum of the wave vectors of the counter-propagating lasers^{16,17,18}. A first pulse at time $t = 0$ splits the atoms into a coherent superposition of hyperfine states gg' with the photon recoil velocity yielding a spatial separation of the two wave packets. A time interval T later the two wave packets are redirected toward each other by a second laser pulse thereby exchanging the internal states g and g' . Finally a time interval T' later the atomic beams recombine and a third pulse is applied. After this pulse the interference pattern in the ground and excited states is measured.

The calculation of the phase shift $\Delta\varphi$ of the atomic interferometer in the presence of a gravitational field proceeds in several steps^{16,22,17}. The first contribution to the phase shift comes from the free propagation of the atoms in the two paths. Since atom interferometers are close to the classical regime, a path integral approach is very appropriate as it reduces to a calculation of integrals along classical paths for a Lagrangian which is at most quadratic in position z and velocity \dot{z} , i.e. is of the general type¹⁷

$$L[z, \dot{z}] = a(t) \dot{z}^2 + b(t) \dot{z}z + c(t) z^2 + d(t) \dot{z} + e(t) z + f(t), \quad (1)$$

where $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$ and $f(t)$ denote some arbitrary functions of time t . The phase shift due to the free propagation of the atoms is given by the *classical* action

$$S_{\text{cl}}(z_T, T; z_0, 0) = \int_0^T dt L[z_{\text{cl}}(t), \dot{z}_{\text{cl}}(t)], \quad (2)$$

where the integral extends over the classical path $z_{\text{cl}}(t)$ obeying the Lagrange equations, with boundary conditions $z_{\text{cl}}(0) = z_0$ and $z_{\text{cl}}(T) = z_T$. Thus the phase difference due to the free propagation of the atoms in the interferometer is equal to the difference of classical actions in the two paths,

$$\Delta\varphi_S \equiv \frac{\Delta S_{\text{cl}}}{\hbar} = \frac{1}{\hbar} \oint dt L[z_{\text{cl}}, \dot{z}_{\text{cl}}], \quad (3)$$

where we use the notation $\oint d\tau$ to mean the difference of integrals between the two paths of the interferometer, assumed to form a close contour.

Theorem^{16,22,17,18,19,20,21,11}. For any quadratic Lagrangian of the form (1) the difference of classical actions in the interferometer, and therefore the phase shift due to the free propagation of the atoms, reduces to the contribution of the change of internal states g and g' , thus^a

$$\Delta S_{\text{cl}} = E_{gg'}(T - T'), \quad (4)$$

where the internal energy change is denoted by $E_{gg'} \equiv E_{g'} - E_g$. (In particular, when the interferometer is symmetric, which will be the case for a closed Mach-Zehnder geometry, and for $T = T'$, we get exactly $\Delta S_{\text{cl}} = 0$.)

One calculates the classical trajectories of the wave packets in the two arms using the equations of motion of massive test bodies deduced from the classical Lagrangian and the known boundary conditions (position and momenta) of the wave packets. In the case of the general quadratic Lagrangian (1) the equations of motion read

$$\frac{d}{dt} [2a(t)\dot{z}] = [2c(t) - \dot{b}(t)]z + e(t) - \dot{d}(t). \quad (5)$$

Then, one calculates the difference in the classical action integrals along the two paths. Denoting by $z_1(t)$ and $z_3(t)$ the classical trajectories between the laser interactions in the upper path, and by $z_2(t)$ and $z_4(t)$ the trajectories in the lower path, we have

$$\Delta S_{\text{cl}} = \int_0^T (L[z_1, \dot{z}_1] - L[z_2, \dot{z}_2]) dt + \int_T^{T+T'} (L[z_3, \dot{z}_3] - L[z_4, \dot{z}_4]) dt + E_{gg'}(T - T'), \quad (6)$$

where the integrals are carried out along the classical paths calculated in the first step. We have taken into account the changes in energy $E_{gg'}$ between the hyperfine ground states g and g' of the atoms in each path. These energies will cancel out from the two paths provided that T' is equal to T , which will be true for a Lagrangian in which we neglect gravity gradients¹⁸.

We now show that the two action integrals in (6) cancel each other in the case of the quadratic Lagrangian (1). This follows from the fact that the difference between the Lagrangians $L[z_1(t), \dot{z}_1(t)]$ and $L[z_2(t), \dot{z}_2(t)]$, which are evaluated at the same time t but on two different trajectories $z_1(t)$ and $z_2(t)$, is a total time-derivative when the Lagrangians are “on-shell”, i.e. when the two trajectories $z_1(t)$ and $z_2(t)$ satisfy the equations of motion (5). To prove this we consider the difference of Lagrangians $L_1 - L_2 \equiv L[z_1, \dot{z}_1] - L[z_2, \dot{z}_2]$ on the two paths, namely

$$L_1 - L_2 = a(\dot{z}_1^2 - \dot{z}_2^2) + b(\dot{z}_1 z_1 - \dot{z}_2 z_2) + c(z_1^2 - z_2^2) + d(\dot{z}_1 - \dot{z}_2) + e(z_1 - z_2). \quad (7)$$

We re-express the first contribution $a(\dot{z}_1^2 - \dot{z}_2^2)$ thanks to an integration by parts as $a(\dot{z}_1^2 - \dot{z}_2^2) = \frac{d}{dt}[a(z_1 - z_2)(\dot{z}_1 + \dot{z}_2)] - (z_1 - z_2)\frac{d}{dt}[a(\dot{z}_1 + \dot{z}_2)]$. The second term is then simplified by means of the sum of the equations of motion (5) written for $z = z_1$ and $z = z_2$. In addition we also integrate by parts the second and fourth contributions in (7) as $b(\dot{z}_1 z_1 - \dot{z}_2 z_2) = \frac{d}{dt}[\frac{1}{2}b(z_1^2 - z_2^2)] - \frac{1}{2}\dot{b}(z_1^2 - z_2^2)$ and $d(\dot{z}_1 - \dot{z}_2) = \frac{d}{dt}[d(z_1 - z_2)] - \dot{d}(z_1 - z_2)$. Summing up the results we obtain

$$L_1 - L_2 = \frac{d}{dt} \left[(z_1 - z_2) \left(a(\dot{z}_1 + \dot{z}_2) + \frac{1}{2}b(z_1 + z_2) + d \right) \right]. \quad (8)$$

Since the difference of Lagrangians is a total time derivative the difference of action functionals in (6) can be immediately integrated. Using the continuity conditions at the interaction points with the lasers (9), which are

$$z_1(0) = z_2(0), \quad (9a)$$

$$z_1(T) = z_3(T), \quad (9b)$$

$$z_2(T) = z_4(T), \quad (9c)$$

$$z_3(T + T') = z_4(T + T'), \quad (9d)$$

^aRigorously, in this equation the time interval should be a proper time interval.

and are appropriate to a closed-path interferometer which closes up at time $T + T'$, we obtain

$$\Delta S_{\text{cl}} = a(T)[z_1(T) - z_2(T)]\left[\dot{z}_1(T) + \dot{z}_2(T) - \dot{z}_3(T) - \dot{z}_4(T)\right] + E_{gg'}(T - T'). \quad (10)$$

Next we apply the boundary conditions in velocities which are determined by the recoils induced from the interactions with the lasers. We see that once we have imposed the closure of the two paths of the interferometer, only the recoils due to the second pulse at the intermediate time T are needed for this calculation. These are given by

$$\dot{z}_1(T) - \dot{z}_3(T) = +\frac{\hbar k}{m}, \quad (11a)$$

$$\dot{z}_2(T) - \dot{z}_4(T) = -\frac{\hbar k}{m}, \quad (11b)$$

where k is the effective wave vector transferred by the lasers to the atoms. This readily shows that the first term in (10) is zero for any quadratic Lagrangian hence $\Delta S_{\text{cl}} = E_{gg'}(T - T')$. ■

Finally, one calculates the contribution to the phase shift due to the light phases of the lasers. These are obtained using the paths calculated previously and the equations of light propagation, with the light acting as a “ruler” that measures the motion of the atoms. The phase difference from light interactions $\Delta\varphi_\ell$ is a sum of terms given by the phases ϕ of the laser light as seen by the atom, i.e. $\phi(z, t) = kz - \omega t - \phi_0$ where k , ω and ϕ_0 are the wave vector, frequency and initial phase of the laser in the frame of the laboratory, and evaluated at all the interaction points with the lasers^{16,18}. Finally the total phase shift measured in the atom interferometer is

$$\Delta\varphi = \omega_{gg'}(T - T') + \Delta\varphi_\ell, \quad (12)$$

and depends only on the internal states g and g' through $\omega_{gg'} = E_{gg'}/\hbar$, and the light phases which measure the free fall trajectories of the atoms. At the Newtonian approximation in a uniform gravitational field g , the interferometer is symmetric, $T' = T$, and one finds¹⁶

$$\Delta\varphi = \Delta\varphi_\ell = k g T^2. \quad (13)$$

This clearly shows that the atom interferometer is a *gravimeter* (or accelerometer): It measures the acceleration g of atoms with respect to the experimental platform which holds the optical and laser elements. With k and T known from auxiliary measurements, one deduces the component of g along the direction of k . If the whole instrument was put into a freely falling laboratory, the measured signal $\Delta\varphi$ would vanish.

The result for the final phase shift (12) or (13) is valid whenever the result (4) holds, i.e. in all theories of gravity defined by a single (quadratic) Lagrangian and consistent with the principle of least action. In such theories the Feynman path integral formulation of quantum mechanics remains valid, and a coherent analysis of atom interferometry experiments is possible. Most alternative theories commonly considered belong to this class which encompasses a large number of models and frameworks¹. It includes for example most non-metric theories, some models motivated by string theory²³ and brane scenarios, some general parameterized frameworks such as the energy conservation formalism^{24,25}, the $\text{TH}\varepsilon\mu$ formalism²⁶, and the Lorentz violating standard model extension (SME)^{27,28}. In all such theories the action-phase shift of the atom interferometer is zero (in particular the Compton frequency of the atom is irrelevant). Because there is no way to disentangle the gravitational redshift from the Doppler shift, we conclude that the recent proposal¹² is invalidated.

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