ULTRA-HIGH RESOLUTION LASER SPECTROSCOPY OF ATOMS AS A PROBE OF GRAVITATIONAL FIELDS INCLUDING GRAVITATIONAL RADIATION

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It has been suggested that the non-linear response of an ensemble of independent two-level atoms (or molecules) submitted to a strong monochromatic electromagnetic wave at ω_1 and probed by a second collinear wave at ω_2 could be a sensitive detector of gravitational waves [1].

The Einstein equivalence principle suggests various quasi-newtonian approaches to this problem which use different frames of reference. However they did not seem to agree on the expected orders of magnitude of the effect [2].

Our goal has been to check these various calculations, to reconcile the points of view and to provide a general framework to treat non-linear optical phenomena in the presence of a gravitational field.

If one interprets the gravitational field within the context of newtonian physics (as generating an acceleration $\vec{a}(\vec{x},\vec{v},t)$), the following equation can be shown to hold, both for the Wigner function and for the density matrix operator when the motion is treated classically (quantum recoil effects can be incorporated following [3]).

$$(1)\left\{\frac{\partial}{\partial t}+\vec{v}.\vec{\nabla}+\vec{a}.\vec{\nabla}_{\vec{v}}\right\}\rho_{\alpha\beta}(\vec{x},\vec{v},t) = \lambda_{\alpha\beta}(\vec{x},\vec{v},t)-(i\,\omega_{\alpha\beta}+\gamma_{\alpha\beta})\rho_{\alpha\beta}+\frac{1}{i\hbar}\sum_{\gamma}\{V_{\alpha\gamma}\rho_{\gamma\beta}-\rho_{\alpha\gamma}V_{\gamma\beta}\}$$

One can use a Liouville space notation for the internal degrees of freedom, then $\rho_{\alpha\beta}$ appears as a vector $|\rho(\vec{x},\vec{v},t)\rangle\rangle$:

$$(2)\left\{\frac{\partial}{\partial t}+\vec{v}.\vec{\nabla}+\vec{a}.\vec{\nabla}_{\vec{v}}\right\}|\rho\rangle\rangle = |\Lambda\rangle\rangle - i\,(L_0+i\,R)|\rho\rangle\rangle - iL_{int}|\rho\rangle\rangle$$

In these equations $\vec{v}$ is the velocity ; the pumping source term is represented by $\lambda_{\alpha\beta}$ in eq.(1) and $|\Lambda\rangle\rangle$ in equ. (2) ; the internal hamiltonian is associated with $\omega_{\alpha\beta}=(E_\alpha-E_\beta)/\hbar$ or L_0 ; $-\gamma_{\alpha\beta}$ and R represent the relaxation. The coupling between the dipole moment $\vec{\mu}$ and the electric field of the lasers $\vec{\mathcal{E}}$ is taken into account through the last terms of equ. (1) and (2) where $V_{\alpha\beta}(\vec{x},t) = -\vec{\mu}_{\alpha\beta}\cdot\vec{\mathcal{E}}(\vec{x},t)$.

An integral form of equations (1) or (2) can be written :

$$(3)\quad |\rho(t)\rangle\rangle = |\rho^0(t)\rangle\rangle - i\int_{-\infty}^{t}dt'\;U(t,t')\,L_{int}\,|\rho(t')\rangle\rangle$$

$\mathcal{U}(t,t')$ is the evolution operator : $\mathcal{U} = \mathcal{C}\exp\{-\int_{t'}^{t}(iL_0 - R + \vec{v}\cdot\vec{\nabla} + \vec{a}\,\vec{\nabla}_{\vec{v}})\,dt''$ where $\mathcal{C}$ denotes the chronological product. $|\rho^0\rangle\rangle$ is the density operator in the absence of laser fields.

Equation (3) can be easily iterated up to any desired order in the strength of the electromagnetic field. Then a perturbative expansion of the solution can be obtained as soon as $|\rho^0\rangle\rangle$ is known.

Two different methods have been used to study the response of the detector. First we have expanded $\mathcal{U}(t,t')$ in a perturbation series with respect to $\vec{a}$, this is the basis of the calculations of reference [1 , 4]. Recently we have used a Magnus expansion [5] of operator $\mathcal{U}$ in the case where $\vec{a}$ depends only on time, which is an excellent approximation ; this work enabled us to reconcile the different points of view.

The main results are the following : Let us consider a first experiment where the atoms undergo no acceleration, where $|\rho^0\rangle\rangle$ is given and where the light sources have the acceleration $-\vec{a}$.

- Let us consider a second experiment where the atoms have the acceleration $\vec{a}$ and where the lasers are not accelerated. These two experiments were shown to be equivalent, provided a suitable change in $|\rho^0\rangle\rangle$ i.e. in $|\Lambda\rangle\rangle$ which is just a change of coordinates from a rest frame to an accelerated one. The latter point is crucial and explains the different orders of magnitude obtained when calculating the linear absorption terms. However, for the non-linear absorption, when the approximation of an infinite Doppler width can be applied, the results do not depend on the frame where $|\rho^0\rangle\rangle$ is given.

The consistency of the two newtonian-like approaches to the problem is most clearly understood within the context of a covariant formalism which has been elaborated at the same time.

The basic equation is

$$i\hbar\, K\{\rho-\rho^0\}_{\alpha\beta} = \sum_{\gamma} V_{\alpha\gamma}\,\rho_{\gamma\beta} - V_{\gamma\beta}\,\rho_{\alpha\gamma} \tag{4}$$

with $K = u^\mu\partial_\mu - \Gamma^{i}_{\rho\sigma}u^\rho u^\sigma \frac{\partial}{\partial u^i}$; $\rho,\mu = 0123$; $i = 123$

$$V_{\alpha\beta} = \mu^{\hat{i}}_{\alpha\beta}\, e^{\rho}_{\hat{i}}\, F_{\rho\lambda}\, u^\lambda$$

where u^μ is the 4-velocity, Γ the connection, F the Maxwell tensor and $e^\rho_{\hat{\sigma}}$ a tetrad which is equal to δ^ρ_σ in the rest frame of the atom before the arrival of the gravitational wave and which undergoes a parallel transport when the gravitational wave is present.

Equation (1) to (4) are solved by iterating their integral expression and using a diagrammatic scheme [3], which is manifestly covariant in the case of equation (4).

As an example let us consider the non-linear term corresponding to

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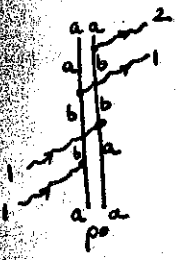

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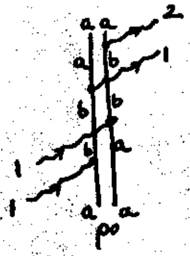

the diagram.

For atoms produced in a Maxwellian velocity distribution one finds the power absorbed per unit volume from the probe laser beam at the distance z from the source and for $\omega_2 = \omega_1 + \xi$

$$\frac{dW_2}{dv} = \hbar\omega_2 \frac{\sqrt{\pi}}{u} n_a^0 \, \Omega_2 (\Omega_1)^3 \, h z \, \mathrm{Im}.\, e^{i\varphi} \left\{ \frac{1}{2\gamma_a\gamma_b} - \frac{1}{2\gamma_{ba}+i\xi}\,\frac{1}{\gamma_b} - \frac{1}{2\gamma_{ba}+i\xi}\cdot\frac{1}{\gamma_b+i\xi} + \frac{1}{2\gamma_{ba}+2i\xi}\cdot\frac{1}{\gamma_b+i\xi} \right\}$$

u is the most probable velocity, n_a^0 is the population of level a, Ω are Rabi pulsations [3] , φ is the phase difference between the laser fields. The acceleration generated by the gravitational wave has been taken as

$$a = \frac{hz}{2}\,\xi^2 \cos \xi t \, .$$

This result is equivalent to previous ones although obtained by a much simpler diagrammatic approach than in [4] .

As a conclusion we emphasize the fact that the results obtained yield a covariant consistent picture of non linear optics in a gravitational field and provide a tool for studying accelerated atomic systems within the frame-work of newtonian physics.

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