Analysis and proposals of the Académie des Sciences working group concerning the redefinition of the unit of mass

The working group of the Académie des Sciences "Unités de base et constantes fondamentales" has carried out a critical analysis of the current system of units (the International System of Units, SI) in the light of our present knowledge of physics and chemistry. The aim was, in particular, to contribute to the present debate on the redefinition of the kilogram on a more perennial and more universal base than the present definition.

A consensus was reached on the need to place this analysis and redefinition within as general a framework as possible, namely on those pillars of modern physics, quantum mechanics and general relativity.

Already from 1923, Louis de Broglie associates with every mass, M, oscillatory phenomena of frequency ν_M such that:

$$h\nu_M = Mc^2 \qquad (1)$$

where h is the Planck constant and c is the speed of light in vacuum.

As this frequency ν_M corresponds, noting the factor c, to the reciprocal Compton wavelength, it will be designated in what follows by the term "de Broglie-Compton frequency".

This association between mass and frequency finds its most general expression in the present formulation of quantum mechanics. In fact, the dimensionless quantum phase (after division by 2π radians) associated with every path in space-time of a particle of mass M is given by the corresponding action divided by the quantum of action h. This phase is written, neglecting signs, as the product of the proper time τ along the path under consideration, and the frequency ν_M. It should be noted that the mass like the proper time is a relativistic invariant.

The de Broglie-Compton frequency of a microscopic object, such as an atom, is today directly measurable by an atom interferometer at the level of a few parts in 10^8 (it can also be obtained indirectly from the Rydberg constant and the fine structure constant). That of a macroscopic object such as the international prototype of the kilogram is also, in principle, accessible at the same level of accuracy through the watt balance thanks to the Josephson and quantum-Hall effects. The ratio of these two de Broglie-Compton frequencies is given by the Avogadro number, whose determination from the properties of a silicon sphere is the goal of an international collaborative project. The relative uncertainty of this determination is about 10^{-7}, but it allows a determination of the de Broglie-Compton frequency of the kilogram prototype with a relative uncertainty of only 10^{-6} for reasons as yet poorly understood, perhaps related to the isotopic composition of the silicon. A new determination using a sphere of isotopically enriched silicon is underway.

The critical analysis carried out showed that two objectives can be pursued:

- Redefine the kilogram from the mass of an elementary particle such as the electron. This definition brings in only the Avogadro number but it should however be consistent with watt balance measurements for the de Broglie-Compton frequency of the kilogram prototype.

- Redefine the unit of mass from the unit of time fixing the conversion factor h/c^2 which appears in the fundamental equation (1). This has the advantage of reducing the number of independent base units as was the case when the metre was redefined. This implies fixing the de Broglie-Compton frequency of the kilogram prototype such as it is measured by the watt balance, which effectively fixes the value of the Planck constant, h, the choice endorsed by the working group. This de Broglie-Compton frequency can also be obtained from the product of the de Broglie-Compton frequency of an atom and the Avogadro number, the two not being able to be fixed independently without touching the unit of time.

These two objectives are thus very different but each of them implies a coherence between the measurements made by the watt balance and those deduced from the Avogadro project completed by atomic interferometry, a coherence that exists today only at the level of a part in 10^6 taking into account the uncertainties of the two methods.

It is, nevertheless, possible to fix the Planck constant in the near future with a number of advantages (in particular to reduce the uncertainties of other fundamental constants) and to use the present kilogram prototype as one of the means of realization in the context of a *mise-en-pratique* of the new definition.

A definition that is simple and solidly founded from the point of view of theory could be the following:

"The kilogram is the unit of mass; it is the mass of a body whose de Broglie-Compton frequency is equal to $[(299\ 792\ 458)^2/(6.626\ 069\ 3 \times 10^{-34})]$ hertz exactly."

This definition has the effect of fixing the value of the Planck constant, *h*, to be $6.626\ 069\ 3 \times 10^{-34}$ J s exactly.

This definition should be accompanied by a detailed explanation and a recommended *mise-en-pratique* for its realization.

This proposition to fix the unit of mass by fixing the value of the Planck constant makes even more important and urgent the continued support of research on the watt balance and on any other connected technique for the realization of the new definition.

The working group also thought about the possibility of redefining the kelvin by fixing the value of the Boltzmann constant. A number of methods currently being studied would allow such a step to be taken within the foreseeable future. Such a step would be scientifically coherent with that of fixing the value of the Planck constant and is thus recommended by the working group.

The working group is also currently interested in the possibility of redefining the electromagnetic units by fixing the absolute value of the electronic charge e. Such a definition is also linked to the bases of quantum mechanics. In fact, the quantum phase associated with an electric potential V is given by the integral over time of $eV/\hbar$ and, once the value of h has been fixed, the conversion factor e/h would be known exactly. The working group is continuing in this direction.

Relating all base units to the unit of time is a conceptually seductive aim, which presents the advantage of opening up to all metrology the exceptional quality of today's atomic clocks.

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