A few comments on the reform of the SI

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On the unit of mass:

One can have a fascinating discussion on the question of whether quantum mechanics applies or not at the macroscopic scale of the kilogram and on the real significance of the appearance of the Planck constant in the watt balance formula. This debate has already started and must continue. Whatever comes out, we are all already persuaded that the mass of a macroscopic object is the sum of that of all its microscopic constituents and of a weak approximately calculable interaction term. This hypothesis is implicit in both possible new definitions of the unit of mass. The concept of mass must be identical at all scales and mass is an additive quantity in the non-relativistic limit. There is no doubt also that, at the atomic scale, mass is directly associated with a frequency via the Planck constant. This frequency can be measured directly for atoms and molecules even though it is quite a large frequency. Measurements of mc^2/h are presently performed with a relative uncertainty much better than 10^{-8}. By **additivity** the link between a macroscopic mass and a frequency is thus unavoidable. If one accepts to redefine the unit of mass from that of a microscopic particle such as the electron, then the link with the unit of time is *ipso facto* established with a relative uncertainty much better than 10^{-8}. Both units are *de facto* linked by the Planck constant to better than 10^{-8}. It seems difficult to ignore this link and not to inscribe it in the formulation of the system of units, especially since it leads to a reduction of the number of independent units.

Another extremely important point is that mass is a relativistic invariant. It should thus never be associated with the frequency of a photon field, which transforms as the time component of a 4-vector in reference frame changes. The de Broglie-Compton frequency is a clock **proper** frequency, Lorentz scalar, equal **by definition** to mc^2/h.

On the electrical units:

The working group of the Académie des Sciences has not tried to impose a system rather than another. It has simply tried to analyze the pros and cons of the present system based upon a fixed value of the vacuum permeability μ_0 compared to a new system in which it is the value of the positron charge *e* which would be fixed.

In the present SI, the values of μ_0 and ε_0 are fixed and thus the propagation properties of the electromagnetic field in the vacuum are also fixed: propagation velocity $c_0 = 1/\sqrt{\varepsilon_0 \mu_0}$, vacuum impedance $Z_0 = \sqrt{\mu_0 / \varepsilon_0}$, as well as the expressions of electric and magnetic energy densities

$$\varepsilon_0 E^2 / 2 \text{ and } \mu_0 H^2 / 2$$

This system is perfectly adapted to the propagation of light in vacuum: no charges but also no ether. Let us now introduce charges. In this system there is a natural unit of electric charge, the so-called Planck charge $q_P = \sqrt{2\varepsilon_0 hc}$ and the ratio of the positron charge *e* to this charge is simply $\sqrt{\alpha}$, dimensionless constant imposed by nature, extraordinalily well-known today since the present uncertainty is only $0.7\text{x}10^{-9}$. In modern physics, the electric charge of a particle is fundamentally an angle thus without dimension. A dimensionless charge equal to $\sqrt{\alpha}$ for the positron is the choice adopted by almost all field-theory experts. The electron is an excitation of the vacuum. It is a complex object, whose ultimate structure is not known. Its bare charge is infinite and it requires a renormalization process to account for the experimentally observed charge. If one chooses to fix this renormalized charge *e,* one will unfortunately lose the consistency in the free propagation properties of electromagnetic fields in the vacuum, since μ_0 and ε_0 will be determined by the measured value of α. The uncertainty of this measurement is therefore transferred to the vacuum properties. One reintroduces a kind of ether, which satisfies some theoreticians who see there the possibility to introduce hypothetical scalar fields (dilatons) suggested by string theory. Is there any other advantage for electrical measurements ?

It clarifies future issues to introduce a specific notation for the approximate theoretical expressions (at the lowest order) of R_{K} and K_{J} :

$$R_{\mathrm{K}}^{(0)} = h/e^2 \quad K_{\mathrm{J}}^{(0)} = 2e/h$$

in order to distinguish them from the true experimental constants R_{K} and K_{J} which are related to the previous ones by:

$$R_{\mathrm{K}} = R_{\mathrm{K}}^{(0)}(1+\varepsilon_{\mathrm{K}}) \quad K_{\mathrm{J}} = K_{\mathrm{J}}^{(0)}(1+\varepsilon_{\mathrm{J}})$$

Fix *h* and *e* would fix the constants $R_{\mathrm{K}}^{(0)}$ and $K_{\mathrm{J}}^{(0)}$ but not R_{K} and K_{J} which would keep and will always keep an uncertainty. This uncertainty is not that related to the determination of *e* and *h* in the SI but to our lack of knowledge of the correction terms to the expressions of R_{K} and K_{J}. Let us recall that the present estimate of the value of ε_{K} is of the order of 2.10^{-8} and that of ε_{J} of the order of 2.10^{-7} with important uncertainties. The fact that the universality of these constants has been demonstrated to a much better level simply suggests that possible corrections would involve other combinations of fundamental constants: functions of α, mass ratios, …

The hydrogen spectrum provides an illustrating example of a similar situation. The energy of the levels of atomic hydrogen is given to the lowest order by Bohr formula, which can also be derived through a topological argument. Nevertheless there are many corrections to this first term involving various fundamental constants. It is not because the spectrum of hydrogen is universal that we may ignore these corrections and restrict ourselves to Bohr formula.

Let us not forget that Cooper pairs are not elementary particles. They exist through a coupling with a lattice and the two electrons may have all kinds of other interactions.

If there remains a large uncertainty for R_{K} and K_{J} and if in addition vacuum properties acquire a new uncertainty, there seems to be no real advantage in fixing the value of *e* rather than that of μ_0 .

It is generally agreed that the “mise-en-pratique” of the present definition of the ampere by an electrodynamometer is not any more satisfactory. But the calculable Lampard-Thompson capacitor fortunately takes over to materialize ε_0 and Z_0 probably at the level of 10^{-8} in the near future. A reformulation of the present definition could be proposed to take into account this new “mise-en-pratique” of electrical properties of the vacuum.

It is thanks to the Lampard that we shall be able to continue testing the expression of R_{K} by comparing R_{K} / Z_0 and $1/(2\alpha)$ and be in a position to use the quantum Hall effect to obtain a secondary representation of the ohm. On the other hand, the validation of K_{J} appears to be possible only through a better determination of the proton (or helion) gyromagnetic ratio or from the watt balance itself.

On the unit of temperature:

The Boltzmann constant comes into play at the microscopic level through its ratio to the Planck constant and at the macroscopic level through its product by the Avogadro number. Any future redefinition of the kelvin should take into account one of these associations, according to the future definition of the unit of mass.

On the definition of the mole:

If the mole is not any more directly connected to 12 grams of carbon, its definition amounts to define an arbitrary number and this number cannot be considered as a fundamental constant of nature. It is only if the mole remains defined by 12 grams of carbon that it rests on a true physical constant. This constant has to be determined experimentally if the unit of mass is defined by fixing the Planck constant.