Does an atom interferometer test the gravitational redshift at the Compton frequency ?

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Abstract

Atom interferometers allow the measurement of the acceleration of freely falling atoms with respect to an experimental platform at rest on Earth's surface. Such experiments have been used to test the universality of free fall by comparing the acceleration of the atoms to that of a classical freely falling object. In a recent paper, Müller, Peters and Chu [A precision measurement of the gravitational redshift by the interference of matter waves, Nature **463**, 926-929 (2010)] argued that atom interferometers also provide a very accurate test of the gravitational redshift (or universality of clock rates). Considering the atom as a clock operating at the Compton frequency associated with the rest mass, they claimed that the interferometer measures the gravitational redshift between the atom-clocks in the two paths of the interferometer at different values of gravitational potentials. In the present paper we analyze this claim in the frame of general relativity and of different alternative theories. We show that the difference of "Compton phases" between the two paths of the interferometer is actually zero in a large class of theories, including general relativity, all metric theories of gravity, most non-metric theories and most theoretical frameworks used to interpret the violations of the equivalence principle. Therefore, in most plausible theoretical frameworks, there is no redshift effect and atom interferometers only test the universality of free fall. We also show that frameworks in which atom interferometers would test the redshift pose dreadful problems, such as (i) violation of the Schiff conjecture, (ii) violation of the Feynman path integral formulation of quantum mechanics and of the principle of least action for matter waves, (iii) violation of energy conservation, and more generally (iv) violation of the particle-wave duality in quantum mechanics. Standard quantum mechanics is no longer valid in such frameworks, so that a consistent interpretation of the experiment would require an alternative formulation of quantum mechanics. As such an alternative has not been proposed to date, we conclude that the interpretation of atom interferometers as testing the gravitational redshift is unsound.

I. INTRODUCTION

A. Motivation

General relativity (GR) has historically been underpinned by distinct types of experimental measurements, which have tested the geometric nature of the theory as well as the specific field equations of GR. In particular, the four "classical tests" are still carried out today with continuously improving accuracy. The first one is the observation of the perihelion advance of planets, for example Mercury, which had been known by Le Verrier in 1845, and explained by general relativity in 1915 (see [1] for a review). The second is the bending of light by the Sun, which was predicted by general relativity and first measured by Eddington in 1919 during a solar eclipse [2]. Another test is the relativistic time delay of radio waves grazing the Sun during their round trip to Mercury and which was computed and measured by Shapiro in 1964 [3].

The gravitational redshift, also considered as a "classical test", is actually a test of one facet of the equivalence principle. The weak equivalence principle has been verified with high precision using torsion balances [4–6] and the Lunar laser ranging [7]. The gravitational redshift was predicted by Einstein but not observed for a long time [2]. It became observable with the advent of high precision quantum spectroscopy and was first measured in 1960 by Pound and Rebka [8] (see also [9]), who used gamma ray spectroscopy of the radiation emitted and absorbed by ^{57}Fe nuclei. The emitter and absorber were placed at the top and bottom of a 22.5 m high tower at Harvard and the frequency difference predicted by GR was measured with about 1% uncertainty. Since then, the initial prediction of a gravitational redshift of solar spectral lines emitted at the Sun's surface has also been measured [10].

In a test of the gravitational redshift using clocks, one checks that the clock rates are universal — i.e. the relative rates depend only on the difference of gravitational potentials (as determined by the trajectories of massive test bodies) but not on the nature and internal structure of the clocks. In 1976 a hydrogen-maser clock was launched on a rocket to an altitude of 10, 000 km and its frequency compared to a similar clock on ground. This yielded a test of the gravitational redshift with about 10^{-4} accuracy [11]. The European Space Agency will fly in 2013 on the International Space Station the Atomic Clock Ensemble in Space (ACES), including a highly stable laser-cooled atomic clock, which will test (in addition to many other applications in fundamental physics and metrology) the gravitational redshift to a precision of 2×10^{-6} [12]. The gravitational redshift is also tested in null redshift experiments in which the rates of different clocks (based on different physical processes or different atoms) are compared to each other [13, 14].

Quite generally in a modern context, tests of GR measure the difference between the predictions of GR and of some generalized alternative theory or theoretical framework (see [15] for a review). For example, GR may be compared to alternative metric theories of gravitation such as the Jordan-Brans-Dicke scalar-tensor theory [16, 17]. The metric theories may also be compared with non-metric theories of gravity such as the Belinfante-Swihart theory [18]. It has to be kept in mind that the classification and inter-comparison of different tests have then to be defined with respect to the framework used.

In the following we will investigate, whether a gravimeter (or accelerometer) based on atom interferometry, can test the gravitational redshift. Atom interferometers allow one to measure the acceleration of atoms falling in the gravitational field of the Earth (with respect to the experimental platform at rest on Earth). The beam-splitter for atom waves is realized through the interaction of atoms with laser beams resonant with a hyperfine atomic transition. It is known [19–26] that the main contribution to the phase shift in atom interferometers comes from the phase imprinted on the matter wave by the beam splitters. Following earlier suggestions by Bordé [19], the first gravimeter based on atom interferometry was realized by Kasevich and Chu [20]. Before that, the first gravimeter based on neutron interferometry had been realized by Colella, Overhauser and Werner [27] (see [28] for a review).

Atom interferometers have reached high sensitivities in the measurement of the gravitational acceleration [29, 30] and rotational acceleration through the Sagnac effect [31–33]. This yields a very important test of the weak equivalence principle or universality of free fall (UFF) when comparing the free fall of atoms with that of classical macroscopic matter (in practice a nearby freely falling corner cube whose trajectory is monitored by lasers). The relative precision on the test of the UFF is currently 7×10^{-9} [29, 30, 34]. Although it remains less sensitive than tests using macroscopic bodies of different composition which have reached a precision of 2×10^{-13} [6, 7], this UFF test is interesting as it is the most sensitive one comparing the free fall of quantum objects (namely Cæsium atoms) with that of a classical test mass (the corner cube).

B. Overview

In a recent paper, Müller, Peters and Chu [34] (hereafter abbreviated as MPC) proposed a new interpretation of atom interferometry experiments as testing the gravitational redshift, that is also the universality of clock rates (UCR), with a precision 7×10^{-9}, which is several orders of magnitude better than the best present [11] and near future [12] clock tests.

The main argument of MPC (see also the more detailed paper [35]) is based on an analogy between atom interferometry experiments and classical clock experiments. The idea of clock experiments is to synchronize a pair of clocks when they are located closely to one another, and move them to different elevations in a gravitational field. The gravitational redshift will decrease the oscillation frequency of the lower clock relative to the higher one, yielding a measurable phase shift between them. There are two methods for measuring the effect. Either we bring the clocks back together and compare the number of elapsed oscillations, or we measure the redshift by means of continuous exchanges of electromagnetic signals between the two clocks. In both methods one has to monitor carefully the trajectories of the two clocks. For example, in the second method one has to remove the Doppler shifts necessarily appearing in the exchanges of electromagnetic signals.

In the first method, the phase difference between the two clocks when they are recombined together, can be written as a difference of integrals over proper time,

$$\Delta\varphi_{\text{clock}} = \omega \left[\int_{\text{I}} \mathrm{d}\tau - \int_{\text{II}} \mathrm{d}\tau \right] \equiv \omega \oint \mathrm{d}\tau \,. \tag{1.1}$$

The two clocks have identical proper frequency ω. We denote by I and II the two paths (with say I being at a higher altitude, i.e. a lower gravitational potential) and use the notation $\oint \mathrm{d}\tau$ to mean the difference of proper times between the two paths, assumed to form a close contour. The integrals in (1.1) are evaluated along the paths of the clocks, and we may use the Schwarzschild metric to obtain an explicit expression of the measured phase shift in the gravitational field of the Earth.

It is indeed true that the phase shift measured by an atom interferometer contains a contribution which is similar to the clock phase shift (1.1). In this analogy, the role of the clock's proper frequency is played by the atom's (de Broglie-)Compton frequency [36]

$$\omega_{\rm C} = \frac{mc^2}{\hbar}\,, \tag{1.2}$$

where m denotes the rest mass of the atom. However the phase shift includes also another contribution $\Delta\varphi_\ell$ coming from the interaction of the laser light used in the beam-splitting process with the atoms. Thus,

$$\Delta\varphi = \omega_{\rm C} \oint \mathrm{d}\tau + \Delta\varphi_\ell\,. \tag{1.3}$$

Here we are assuming that the two paths close up at the entry and exit of the interferometer; otherwise, additional terms have to be added to (1.3). The first term in (1.3) is proportional to the atom's mass through the Compton frequency and represents the difference of Compton phases along the two classical paths. In contrast, the second term $\Delta\varphi_\ell$ does not depend on the mass of the atoms.

At first sight, the first term in Eq. (1.3) could be used for a test of the gravitational redshift, in analogy with classical clock experiments, cf. Eq. (1.1), and the precision of the test could be very good, because the Compton frequency of the Cæsium atom is very high, $\omega_{\rm C} \approx 2\pi \times 3.0 \times 10^{25}$ Hz. However, as shown in a previous brief reply [37], this re-interpretion of the atom interferometer as testing the UCR is fundamentally incorrect. The clear-cut argument showing that the atom interferometer does not measure the redshift is that the "atom-clock" contribution, i.e. the first term in (1.3), is in fact *zero* for a closed total path [21, 23, 25, 26]. Thus only the second term $\Delta\varphi_\ell$ remains in Eq. (1.3). The final phase shift,

$$\Delta\varphi = \Delta\varphi_\ell = k\,g\,T^2\,, \tag{1.4}$$

depends on the wavevector k of the lasers, on the interrogation time T and on the local gravity g, but is independent of the Compton frequency (1.2).

In the present paper, we expand our previous brief reply and address in detail the involved issues. As we shall detail below, the first term in (1.3) vanishes in GR and in all metric theories of gravity, and in a large class of non-metric test theories which emcompasses most theoretical frameworks used to interpret the violations of the different facets of the equivalence principle [15].

The key point about the result (1.4) is a consistent calculation of the two paths in the atom interferometer and of the phases along these paths, both derived from the same classical action, using in a standard way the principle of least action. At the deepest level, the principle of least action and its use in atom interferometry comes from the Feynman path integral formulation of quantum mechanics or equivalently the Schrödinger equation [21]. We will see in the following that the argument of MPC implies a violation of the principle of least action. The significance and the sensitivity of atom interferometry should then be evaluated in the framework of an alternative theory because it is not consistent to calculate, as MPC do, the dephasing of the interferometer by using the Feynman approach with two different Lagrangians. Instead, a different approach for the description of atom interferometers has to be developed for dealing with the violation of fundamental principles of quantum mechanics.

These investigations lie at the interface between quantum mechanics and gravitational theories. One could wonder whether it is possible to provide clear answers to the question asked in the title of this paper. We shall see below that a number of clear-cut statements and definite conclusions will be obtained, provided the theoretical frameworks are specified clearly and unambiguously. To this aim, we shall consider two general alternative theoretical frameworks.

The first framework, which we call the "modified Lagrangian formalism", consists of a deformation of the Lagrangian of GR [15, 38, 39]. Such a modified Lagrangian formalism is at the basis of the usual interpretation of the various tests of the equivalence principle, including the tests of the UCR and the UFF, in a way consistent with Schiff's conjecture [40], the principle of least action and the basic principles of quantum mechanics. Within this general formalism we will prove that the atom interferometry experiment is not a test of the UCR but only a test of the UFF.

In the second framework, which we call the "multiple Lagrangian formalism", the *motion* of test particles (atoms or macroscopic bodies) obeys the standard GR Lagrangian in a gravitational field, whereas the *phase* of the corresponding matter waves obeys a *different* Lagrangian. The MPC analysis belongs to this framework which raises extremely difficult problems. It violates the Feynman path integral method which is at the basis of the computation of the quantum phase shift, as well as the Schiff conjecture. In particular, the phase shift is derived in a manner which is not consistent with standard quantum mechanics. This derivation would thus require the development of a mathematically sound theory allowing one to go beyond standard quantum mechanics. It is only after this development that the interpretation of atom interferometry experiments as testing the gravitational redshift could be attributed an unambiguous meaning.

At this point, we also want to mention other crucial differences between atom interferometry and clock experiments. In clock experiments the trajectories of the clocks are continuously controlled for instance by continuous exchange of electromagnetic signals. In atom interferometry in contrast, the trajectories of the atoms are not measured independently but theoretically derived from the Lagrangian and initial conditions. Furthermore, we expect that it is impossible to determine independently the trajectories of the wave packets without destroying the interference pattern at the exit of the interferometer.[1]

Another important difference lies in the very notion of a clock. Atomic clocks use the extremely stable energy difference between two internal states. By varying the frequency of an interrogation signal (e.g. microwave or optical), one obtains a resonant signal when the frequency is tuned to the frequency of the atomic transition. In their re-interpretation, MPC view the entire atom as a clock ticking at the Compton frequency associated with its rest mass. But the "atom-clock" is not a real clock in the previous sense, since it does not deliver a physical signal at Compton frequency.

The plan of this paper is as follows. In Sec. II we discuss the prediction of GR, emphasizing its foundations in Secs. II A and II B, and standard interpretation in Sec. II C. In Sec. III we investigate the alternative multiple Lagrangian formalism (Sec. III A) and

[1] Note also that the phase shift (1.1) for clocks is valid in any gravitational field, with any gravity gradients, since it is simply the proper time elapsed along the trajectories of the clocks in a gravitational field. By contrast, the phase shift (1.3) is known only for quadratic Lagrangians (see Sec. II A) and cannot be applied in a gravitational field with large gravity gradients, or more generally with any Lagrangian that is of higher order.

modified Lagrangian formalism (Sec. III B). Our conclusions are summarized in Sec. IV. The case of neutron interferometry is briefly discussed in Appendix A. We provide some estimate of the influence of high-order gravity gradients in Appendix B.

II. GRAVIMETRY USING ATOM INTERFEROMETERS

A. Path integral formalism

Theoretical tools for atom optics and interferometry have been extensively developed [21, 23–26]. Since atom interferometers are close to the classical regime ($S \gg \hbar$), a path integral approach is very appropriate since it reduces to a calculation of integrals along classical paths for Lagrangians which are at most quadratic in position and velocity. The path integral approach to matter wave interferometry is developed in detail in Ref. [21]. Equivalent results have been obtained using non-relativistic or relativistic wave equations of quantum mechanics (see e.g. Ref. [23] for a review). Here we only recall the points relevant to our discussion. We denote by $K(z_b, t_b; z_a, t_a)$ the quantum propagator, that provides the wavefunction of the atom at point (z_b, t_b) given the wavefunction at any initial point (z_a, t_a),

$$\psi(z_b, t_b) = \int \mathrm{d}z_a \, K(z_b, t_b; z_a, t_a) \, \psi(z_a, t_a) \,, \tag{2.1}$$

where the integration is over all initial positions z_a at the given time t_a. In Feynman's formulation the propagator is a sum over all possible paths Γ connecting (z_a, t_a) to (z_b, t_b),

$$K(z_b, t_b; z_a, t_a) = \int \mathcal{D}z_\Gamma(t) \, \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S_\Gamma} \,, \tag{2.2}$$

where $\mathcal{D}z_\Gamma(t)$ is the integration measure over the path $z_\Gamma(t)$, which is such that $z_\Gamma(t_a) = z_a$ and $z_\Gamma(t_b) = z_b$. The phase factor is computed from the action along each path given by

$$S_\Gamma(z_b, t_b; z_a, t_a) = \int_{t_a}^{t_b} \mathrm{d}t \, L\left[z_\Gamma(t), \dot{z}_\Gamma(t)\right] . \tag{2.3}$$

It can be shown [21] that in the case where the Lagrangian is at most *quadratic* in positions z and velocities $\dot{z}$, i.e. is of the general type

$$L\left[z, \dot{z}\right] = a(t) \, \dot{z}^2 + b(t) \, \dot{z} z + c(t) \, z^2 + d(t) \, \dot{z} + e(t) \, z + f(t) \,, \tag{2.4}$$

where $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$ and $f(t)$ denote some arbitrary functions of time t, the propagator takes the simple form

$$K(z_b, t_b; z_a, t_a) = F(t_b, t_a) \, \mathrm{e}^{\frac{\mathrm{i}}{\hbar} S_{\mathrm{cl}}(z_b, t_b; z_a, t_a)} \,. \tag{2.5}$$

Here the function F depends on times t_a, t_b but not on positions z_a, z_b, which is no longer the case when the Lagrangian is more than quadratic [21]. We denote by S_{cl} the *classical* action

$$S_{\mathrm{cl}}(z_b, t_b; z_a, t_a) = \int_{t_a}^{t_b} \mathrm{d}t \, L\left[z_{\mathrm{cl}}, \dot{z}_{\mathrm{cl}}\right] \,, \tag{2.6}$$

where the integral extends over the classical path $z_{\mathrm{cl}}(t)$ satisfying the Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{z}_{\mathrm{cl}}}\right) = \frac{\partial L}{\partial z_{\mathrm{cl}}}\,, \tag{2.7}$$

with boundary conditions $z_{\mathrm{cl}}(t_a) = z_a$ and $z_{\mathrm{cl}}(t_b) = z_b$. One can show that the wave function deduced from (2.1) and (2.5) obeys the Schrödinger equation corresponding to the Hamiltonian H deduced from the Lagrangian L.

Next the result (2.5) is used to obtain the final wavefunction (2.1) in the case where the initial state $\psi(z_a, t_a)$ is a plane wave, i.e.

$$\psi(z_a, t_a) = \frac{1}{\sqrt{2\pi\hbar}}\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}(p_0 z_a - E_0 t_a)}\,. \tag{2.8}$$

Denoting by z_0 the initial position at time t_a such that the initial momentum of the classical path starting from (z_0, t_a) equals the momentum p_0 of the initial plane wave, we obtain

$$\psi(z_b, t_b) = G(t_b, t_a)\,\psi(z_0, t_a)\,\mathrm{e}^{\frac{\mathrm{i}}{\hbar}S_{\mathrm{cl}}(z_b, t_b; z_0, t_a)}\,. \tag{2.9}$$

The function G is another function depending only on times t_a and t_b. The result (2.9) is at the basis of the computation of the phase shift in atom interferometers. We shall from now on change our notation z_0 back into z_a so that (2.9) applies to say the half upper path between (z_a, t_a) and (z_b, t_b) of the interferometer depicted in Fig. 1. We emphasize at this point that this formula is valid only for quadratic Lagrangians of the form (2.4). For more general Lagrangians, the function G a priori depends on the positions z_a and z_b, and the simple formula (2.9) is a priori incorrect.

Evidently, any attempt like the one proposed by MPC to integrate the classical action (2.6) over a path which is different from the classical path, i.e. which does not obey the Lagrange equation (2.7) or is solution of a Lagrange equation with a different Lagrangian, will violate the fundamental Feynman path integral formulation of quantum mechanics. The wave function (2.9) is then no longer a solution of the Schrödinger equation and it becomes inconsistent to use it for the calculation of the phase difference. This inconsistency is inherent to the class of formalisms investigated in Sec. III A, and, as we shall see, is the only way to substantiate the claim made by MPC.

B. Computation of the phase shift

The atomic gravimeter is described in detail in [19, 20, 22, 29] and the general theory of such gravito-inertial devices can be found, for example, in [21, 23, 24]. The schematic view of the atom interferometer showing the two interferometer paths I and II is given by Fig. 1. Here we recall only the main points relevant to our discussion (see the detailed description of the experimental setup in Ref. [29]).

The Cæsium (or some alkali) atoms are optically cooled and launched in a vertical fountain geometry. They are prepared in a hyperfine ground state g. A sequence of vertical laser pulses resonant with a $g \to g'$ hyperfine transition is applied to the atoms during their ballistic (i.e. free fall) flight. In the actual experiments the atoms undergo a two-photon Raman transition where the two Raman laser beams are counter-propagating. This results in a recoil velocity of the atoms, with the effective wave vector transferred to the atoms

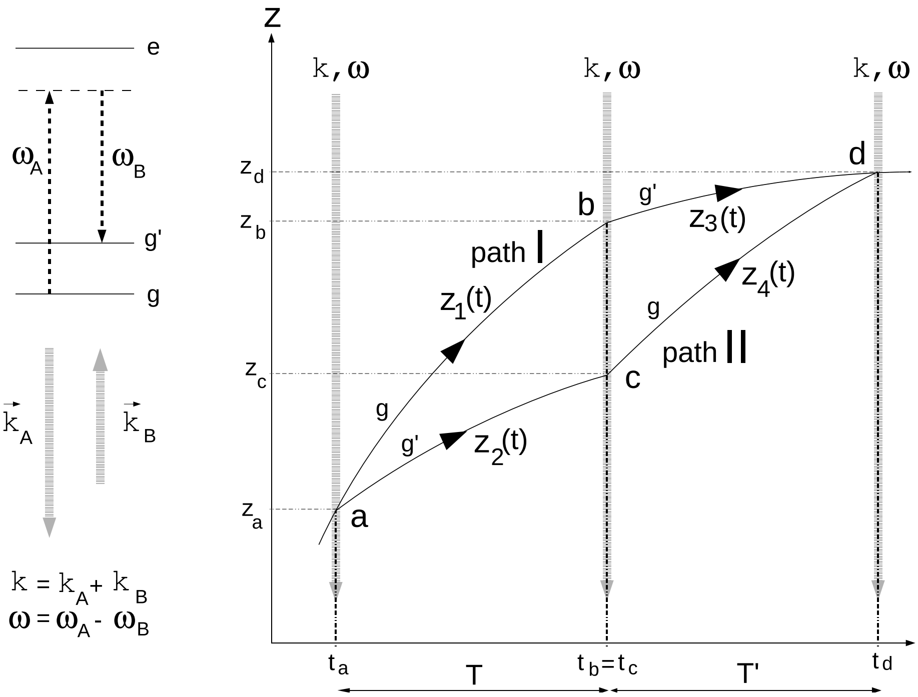

FIG. 1: Space-time trajectories followed by the atoms in the interferometer. Laser pulses occur at times t_a, $t_b = t_c$ and t_d, separated by time intervals T and T'. The two-photon Raman transitions are between two hyperfine levels g and g' of the ground state of an alkali atom.

being $k = k_A + k_B$, where A and B refer to the counter-propagating lasers (see e.g. [21, 22]). A first pulse at time t_a splits the atoms into a coherent superposition of hyperfine states gg' with the photon recoil velocity yielding a spatial separation of the two wave packets. A time interval T later (we denote $T = t_b - t_a = t_c - t_a$; see Fig. 1) the two wave packets are redirected toward each other by a second laser pulse thereby exchanging the internal states g and g'. Finally a time interval T' later (with $T' = t_d - t_b = t_d - t_c$) the atomic beams recombine and a third pulse is applied. After this pulse the interference pattern in the ground and excited states is measured.

The calculation of the phase shift $\Delta\varphi$ of the atomic interferometer in the presence of a gravitational field was done in Refs. [19–21] and proceeds in several steps. The first contribution to the phase shift comes from the free propagation of the atoms in the two paths. Using the result (2.9) of the path integral formalism, and the fact that the function G therein depends on times t_a, t_b but not on positions z_a, z_b, we see that this contribution is equal to the difference of classical actions in the two paths,

$$\Delta\varphi_S = \frac{\Delta S_{\rm cl}}{\hbar}\,. \tag{2.10}$$

To compute it one calculates the classical trajectories of the wave packets in the two arms using the equations of motion of massive test bodies deduced from the classical Lagrangian and the known boundary conditions (position and momenta) of the wave packets. In the case of the general quadratic Lagrangian (2.4) the equations of motion read

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big[2a(t)\dot{z}\Big] = \big[2c(t) - \dot{b}(t)\big]\,z + e(t) - \dot{d}(t)\,. \tag{2.11}$$

Following Fig. 1 we denote by $z_1(t)$ and $z_3(t)$ the classical trajectories between the laser interactions in the upper path I, and by $z_2(t)$ and $z_4(t)$ the trajectories in the lower path II. The boundary conditions in positions appropriate to the closed-path interferometer of Fig. 1 are (for simplicity we set $t_a = 0$)

$$z_1(0) = z_2(0)\,, \tag{2.12a}$$
$$z_1(T) = z_3(T)\,, \tag{2.12b}$$
$$z_2(T) = z_4(T)\,, \tag{2.12c}$$
$$z_3(T+T') = z_4(T+T')\,. \tag{2.12d}$$

The boundary conditions in velocities are determined by the recoils induced from the interactions with the lasers. Actually once we have imposed the boundary conditions (2.12), and in particular that the interferometer closes at some time $T+T'$, only the recoils due to the second pulse at the intermediate time T are needed for this calculation. These are given by

$$\dot{z}_1(T) - \dot{z}_3(T) = +\frac{\hbar k}{m}\,, \tag{2.13a}$$
$$\dot{z}_2(T) - \dot{z}_4(T) = -\frac{\hbar k}{m}\,, \tag{2.13b}$$

where $k = k_A + k_B$ is the effective wave vector transferred by the lasers to the atoms. Second, one calculates the difference in the classical action integrals along the two paths,

$$\Delta S_{\mathrm{cl}} = \int_0^T \Big(L[z_1,\dot{z}_1] - L[z_2,\dot{z}_2]\Big)\mathrm{d}t + \int_T^{T+T'} \Big(L[z_3,\dot{z}_3] - L[z_4,\dot{z}_4]\Big)\mathrm{d}t + \Delta S_{gg'}\,, \tag{2.14}$$

where the Lagrangian $L(z,\dot{z})$ is given by (2.4), and the integrals are carried out along the classical paths calculated in the first step. We have taken into account the changes in energy between the hyperfine ground states g and g' of the atoms in each path. These energies will cancel out from the two paths provided that T' is equal to T, which will be true for a Lagrangian in which we neglect gravity gradients. In the more general case T' will differ from T and there is an extra contribution in (2.14) given by

$$\Delta S_{gg'} = \hbar\,\omega_{gg'}(T - T')\,, \tag{2.15}$$

where we denote the internal energy change by $E_{g'} - E_g = \hbar\,\omega_{gg'}$.

We now prove (see e.g. [21, 23]) that the two action integrals in (2.14) cancel each other in the case of a quadratic Lagrangian of the general form (2.4). This follows from the fact that the difference between the Lagrangians $L[z_1(t),\dot{z}_1(t)]$ and $L[z_2(t),\dot{z}_2(t)]$, which are evaluated at the same time t but on two different trajectories $z_1(t)$ and $z_2(t)$, is a total time-derivative when the Lagrangians are "on-shell", i.e. when the two trajectories $z_1(t)$ and $z_2(t)$ satisfy the equations of motion (2.11). Namely, we find[2]

$$L[z_1,\dot{z}_1] - L[z_2,\dot{z}_2] = \frac{\mathrm{d}}{\mathrm{d}t}\left[(z_1 - z_2)\Big(a(t)(\dot{z}_1 + \dot{z}_2) + \frac{1}{2}b(t)(z_1 + z_2) + d(t)\Big)\right]. \tag{2.16}$$

[2] To prove this we consider the first contribution $a(\dot{z}_1^2 - \dot{z}_2^2)$ in the difference $L_1 - L_2$ and re-express it

Since the difference of Lagrangians is a total time derivative the difference of action functionals in (2.14) can be immediately integrated. Using the continuity conditions at the interaction points with the lasers (2.12), and then using the recoils due to the intermediate laser pulse at time T given by (2.13), yields

$$\Delta S_{\rm cl} - \Delta S_{gg'} = a(T)\big[z_1(T) - z_2(T)\big]\Big[\dot{z}_1(T) + \dot{z}_2(T) - \dot{z}_3(T) - \dot{z}_4(T)\Big] = 0\,. \tag{2.17}$$

Therefore for all quadratic Lagrangians the difference of classical actions in the interferometer, and therefore the phase shift due to the free propagation of the atoms, reduces to the contribution of the change of internal states g and g', thus[3]

$$\Delta\varphi_S = \omega_{gg'}(T - T')\,. \tag{2.18}$$

When the interferometer is symmetric, i.e. $T = T'$ which will be the case when we neglect gravity gradients, we get exactly zero. Equations (2.17) or (2.18) constitute the central theorem to be used for the discussion in the present paper.

In the second step, one calculates the difference in the light phases $\Delta\varphi_\ell$ at the three interactions with lasers. These are obtained using the paths calculated in the first step and the equations of light propagation, with the light acting as a "ruler" that measures the motion of the atoms. For the interferometer described in Fig. 1 the phase difference from light interactions is given by

$$\Delta\varphi_\ell = -\phi(z_a, 0) + \phi(z_b, T) + \phi(z_c, T) - \phi(z_d, T + T')\,. \tag{2.19}$$

Here ϕ denotes the phase of the laser light as seen by the atom, i.e. $\phi(z,t) = kz - \omega t - \phi_0$ where k, ω and ϕ_0 are the wave vector, the frequency and the phase of the laser in the frame of the laboratory. The phase of the laser is evaluated at the interaction points shown in Fig. 1. Finally the total phase shift measured in the atom interferometer is the sum of the two contributions

$$\Delta\varphi = \omega_{gg'}(T - T') + \Delta\varphi_\ell\,, \tag{2.20}$$

depending only on the internal states g and g', and the light phases which measure the free fall trajectories of the atoms.

Here we have assumed that the two paths I and II close up at the entry and exit of the interferometer (see Fig. 1). For the more general case where the paths are not closed, one finds that $\Delta\varphi_S$ is no longer zero, but that $\Delta\varphi_S$ is exactly cancelled by additional terms corresponding to the phase difference of the wave packets at different positions at the interferometer entry and exit [23] (except for the very small terms due to the energy difference $\hbar\omega_{gg'}$).

Expression (2.18) shows that, depending on the particular geometry of the interferometer, one can obtain a sensitivity to the frequency difference $\omega_{gg'}$ of the two internal states. In

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thanks to an integration by parts as $a(\dot{z}_1^2 - \dot{z}_2^2) = \frac{\mathrm{d}}{\mathrm{d}t}[a(z_1 - z_2)(\dot{z}_1 + \dot{z}_2)] - (z_1 - z_2)\frac{\mathrm{d}}{\mathrm{d}t}[a(\dot{z}_1 + \dot{z}_2)]$. The second term is then simplified by means of the sum of the equations of motion (2.11) written for $z = z_1$ and $z = z_2$. In addition we also integrate by parts the second and fourth contributions to $L_1 - L_2$ as $b(\dot{z}_1 z_1 - \dot{z}_2 z_2) = \frac{\mathrm{d}}{\mathrm{d}t}[\frac{1}{2}b(z_1^2 - z_2^2)] - \frac{1}{2}\dot{b}(z_1^2 - z_2^2)$ and $d(\dot{z}_1 - \dot{z}_2) = \frac{\mathrm{d}}{\mathrm{d}t}[d(z_1 - z_2)] - \dot{d}(z_1 - z_2)$. Summing up the results we obtain (2.16).

[3] Rigorously, in this equation the time interval should be a proper time interval.

that sense the interferometer can be viewed as a clock. In fact, atomic clocks are a particular kind of interferometer which has been designed to have a strong sensitivity to $\omega_{gg'}$ [23, 24]. A simple example is an interferometer with only two pulses separated by a time T. In that case one uses either a microwave field or Raman transitions with two co-propagating laser beams so that $k = k_A - k_B$. Then the photon recoil becomes negligibly small and the two paths are separated spatially at $t = T$ by much less than the coherence length of the atoms, allowing them to interfere. The total phase shift is then given by

$$\Delta\varphi = \omega_{gg'}T + \Delta\varphi_\ell \,. \tag{2.21}$$

As mentioned above, all other terms in $\Delta\varphi_S$ are exactly cancelled by corresponding terms arising from the non-closure of the paths. The laser phase shift is to leading order

$$\Delta\varphi_\ell = -\omega T \,. \tag{2.22}$$

One therefore observes interference fringes when varying the frequency of the laser or microwave field, known as Ramsey fringes. That allows locking the laser frequency to the atomic transition frequency $\omega_{gg'}$, which constitutes an atomic clock. We note that the *interference* of the two interferometer arms is required for the operation of the clock, each arm individually cannot be considered as a clock.

If one wanted to use such interferometers (clocks) for a gravitational redshift test one would need two of them placed in different gravitational potentials (e.g. [11, 12]). But, of course such clocks do not run at the Compton frequency but at $\omega_{gg'}$, and thus the corresponding redshift test is not at the Compton frequency. For that to be the case one would need interferometers with the internal energy (mass) *difference* of the particles on the two paths $\hbar\omega_{gg'} = mc^2$. Such matter-antimatter interferometer is orders of magnitude beyond present day technology.

C. Prediction from general relativity

As a specific example, let us consider the prediction from GR, which has been extensively treated in Ref. [23]; here we only present a very basic analysis sufficient for our purposes. The appropriate Lagrangian is derived to sufficient accuracy using the Schwarzschild metric generated by the Earth,

$$L_{\mathrm{GR}}(z,\dot{z}) = -mc^2\frac{\mathrm{d}\tau}{\mathrm{d}t} = -mc^2 + \frac{GMm}{r_\oplus} - mgz + \frac{1}{2}m\dot{z}^2 + \mathcal{O}\left(\frac{1}{c^2}\right) , \tag{2.23}$$

where $\mathrm{d}\tau = \sqrt{-g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu/c^2}$ is the proper time, $r_\oplus$ is the Earth's radius, $g = GM/r_\oplus^2$ is the Newtonian gravitational acceleration, G is Newton's gravitational constant, M is the mass of the Earth, m the mass of the atom, c the speed of light in vacuum, z is defined by $r = r_\oplus + z$ with r the radial coordinate, and $\mathcal{O}(1/c^2)$ denotes a post-Newtonian correction. For simplicity we restrict ourselves to only radial motion, which is sufficient for the arguments presented in this paper. We also neglect systematically the post-Newtonian correction and no longer indicate the remainder term $\mathcal{O}(1/c^2)$.

The equations of motion are deduced from (2.23) using the principle of least action or the Euler-Lagrange equations, and read evidently $\ddot{z} = -g$. Then, when integrating (2.23) along the resulting paths, and calculating the difference as defined by (2.14) one finds

$$\Delta\varphi_S = 0 \,. \tag{2.24}$$

Since the Lagrangian (2.23) is quadratic, this is a particular case of the general result (2.17). Furthermore, in the simple case of a Lagrangian which is linear in z we find that the interferometer is symmetrical, i.e. the interrogation times are equal, $T = T'$. Thus the effect due to the difference of internal energies $E_{g'} - E_g$ is zero in this case.

Evaluating the light phases at the different interaction points one finds easily that (see [21–23, 29] for details)

$$\Delta\varphi_\ell = k\, g\, T^2\,, \tag{2.25}$$

where k is the effective wave vector of the lasers (i.e. $k = k_A + k_B$ where the subscripts A and B refer to the counter-propagating laser beams [22]), and T is the time interval between the laser pulses (see Fig. 1). As a result the total phase shift calculated in GR is

$$\Delta\varphi = k\, g\, T^2\,. \tag{2.26}$$

This shows that the atom interferometer is a gravimeter or accelerometer. The phase shift (2.26) arises entirely from the interactions with the lasers and the fact that the atoms are falling with respect to the laboratory in which the experiment is performed. It is thus proportional to the acceleration g of atoms with respect to the experimental platform which holds the optical and laser elements. With k and T known from auxiliary measurements, one deduces the component of g along the direction of k. If the whole instrument was put into a freely falling laboratory, the measured signal $\Delta\varphi$ would vanish.[4]

The signal (2.26) is actually the same as in the measurement of the free fall of a macroscopic object (corner cube) using lasers. Therefore atomic interferometers can be used for testing the universality of free fall between the atoms and some macroscopic objects. The precision on the test of the UFF between Cæsium atoms and classical objects such as the corner cube is currently 7×10^{-9} [29, 30, 34].

We have stressed that integrating the Lagrangian along the classical paths only provides the correct phase shift when the Lagrangian is at most quadratic in position and velocity [21]. For higher order Lagrangians the full path integrals need to be evaluated. Thus, one might be tempted to use atom interferometry tests to search for redshift effects arising from third and higher order terms in the Lagrangian, but as already mentioned and shown explicitly in e.g. [21] such an approach is incorrect, as the full Feynman integral is required.[5]

III. TESTING THE GRAVITATIONAL REDSHIFT ?

We now address the question of the significance of tests of the gravitational redshift by atom interferometers as described by MPC [34] within the frameworks of different classes of alternative theories or models. Depending on the underlying theory, the Lagrangian will be different, so the equations of motion providing the paths will be different, and the relation between the action integral and the phase shift [cf. Eq. (2.10)] may also be different. In

[4] The result is identical for the phase shift in neutron interferometers under the influence of gravity, as shown in Appendix A.

[5] If one nonetheless calculates the phase shift for a higher-order Lagrangian by integrating only along the classical path, a non zero result is obtained. In Appendix B we estimate the order of magnitude of the effect of including cubic terms in the Lagrangian of GR, which correspond to second-order gravity gradients.

addition we may have to model differently the propagation of light, so the resulting calculated phase shift might also differ. The comparison of the calculated and measured phase shifts then allows to discriminate between different candidate theories. We shall see that the significance of the tests depends on the alternative model we consider.

A. Multiple Lagrangian framework

This formalism constitutes the only framework within which the atom interferometry experiment could be viewed as testing the gravitational redshift or the universality of clock rates (UCR), as proposed by MPC. Within this multiple Lagrangian framework we derive the interferometer phase shift and then we show that this framework raises a number of unacceptable issues related to fundamental principles of quantum mechanics.

The framework is motivated by a search for a possible violation of the local position invariance (LPI) aspect of the Einstein equivalence principle, while supposing that the remaining aspects of the equivalence principle, namely the local Lorentz invariance (LLI) and the weak equivalence principle (WEP), are valid. Thus, in the first place, this framework violates the Schiff conjecture [40]. (See Ref. [15] for a thorough discussion on the different parts of the Einstein equivalence principle and the Schiff conjecture.)

If WEP is valid we can consider the local freely falling frames associated with test bodies, which fall with the universal acceleration g in a gravitational field. In these frames classical clocks will measure a proper time $\mathrm{d}\tau$ which is proportional, because LLI is valid, to the special-relativistic Minkowskian interval $\mathrm{d}s$. However, because LPI is violated, we allow for a dependence on the position of the measured rate. Specifically we introduce a proportionality factor $f(\Phi)$ between $\mathrm{d}\tau$ and $\mathrm{d}s$ built from some anomalous field Φ associated with gravity and depending on position. In an arbitrary frame this means that the proper time $\mathrm{d}\tau$ measured by clocks is proportional to the metric interval $\mathrm{d}s$ and given by

$$\mathrm{d}\tau = f(\Phi)\,\mathrm{d}s = f(\Phi)\sqrt{-g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu/c^2}\,. \tag{3.1}$$

But because WEP is valid the motion of test bodies is unaffected and obtained from the usual variational principle associated with the metric interval $\mathrm{d}s$, i.e. $\delta\int\mathrm{d}s = 0$.

Consider a classical redshift experiment. One observes at point z_1 the light coming from a particular atomic transition occuring at point z_0. The points z_1 and z_0 are at rest in a stationary gravitational field g. The relative difference between the frequency of the light coming from point z_0 and observed at point z_1, with respect to the frequency of the light emitted by the same atomic transition but occuring at point z_1, is

$$Z \equiv \frac{\Delta\tau_1}{\Delta\tau_0} - 1 = \frac{f(\Phi_1)}{f(\Phi_0)}\frac{\sqrt{-g_{00}(z_1)}}{\sqrt{-g_{00}(z_0)}} - 1\,, \tag{3.2}$$

where $\Phi_1 = \Phi(z_1)$ and $\Phi_0 = \Phi(z_0)$. It is convenient to introduce a coefficient ζ linking the gradient of the anomalous field to the local gravitational acceleration by $g = \zeta\,\mathrm{d}\Phi/\mathrm{d}z$ (see [15] for more details). Expanding to first order in $\Delta z = z_1 - z_0$ we can write

$$f = f_0\left[1 + \beta\,\frac{g\Delta z}{c^2}\right]\,, \tag{3.3}$$

where we have defined

$$\beta = \frac{c^2 f_0'}{\zeta\, f_0}\,, \tag{3.4}$$

with f_0 and f_0' denoting f and $\mathrm{d}f/\mathrm{d}\Phi$ evaluated at the value $\Phi(z_0)$. Inserting then (3.3) into (3.2) we find that the parameter β defined in this way measures a possible anomalous deviation from the standard prediction for the redshift, i.e.

$$Z = (1+\beta)\,\frac{g\Delta z}{c^2}\,, \tag{3.5}$$

and we know that β has been tested by redshift experiments with the accuracy 10^{-4} [11].

Usually the parameter measuring redshift violations is denoted α but here it is important to adopt the different notation β used by MPC. Indeed one should recall that the meaning of parameters α or β depends on the theoretical framework in use. The result (3.5) corresponds to our particular formalism, the multiple Lagrangian formalism, and should be contrasted with a similar result (3.19) we shall obtain below in the context of the modified Lagrangian formalism in Sec. III B. In particular we notice that the parameter β in Eq. (3.5) may or may not be "universal", i.e. depending on the nature and internal structure of the clock, in contrast with the parameter $\alpha_X^{(a)}$ in Eq. (3.19) which depends on the type of atom (a) constituting the clock and on some type of internal energy X violating LPI. In the present formalism we do not need to specify if β is universal or not because WEP is valid so we can always test the value of β by measuring g from the free-fall of test bodies (as proposed by MPC). This is different in the modified Lagrangian formalism of Sec. III B where any universal contribution to $\alpha_X^{(a)}$ is unobservable since it can always be absorbed into a re-definition of g.

Let us now apply this formalism to atom interferometry, viewing the atoms in the two paths of the interferometer as classical clocks ticking at the Compton frequency ω_C. In this case z_1 is a generic position of the atom on one of the paths (see Fig. 1) and z_0 denotes an origin located between the paths. Following the analogy with classical clocks discussed in Sec. I B of the Introduction, we can write the phase difference between the two "atom-clocks" for a closed interferometer as[6]

$$\Delta\varphi_S = \omega_\mathrm{C} \oint \mathrm{d}\tau = \omega_\mathrm{C} \oint \left[1 + \beta\,\frac{g\Delta z}{c^2}\right] \mathrm{d}s\,. \tag{3.6}$$

On the other hand the trajectories of the atoms are the usual geodesics of space-time, such that $\delta \int \mathrm{d}s = 0$. As we have proved in Sec. II B, in the case of a quadratic Lagrangian and a symmetric closed interferometer, the integral of $\mathrm{d}s$ is zero, so one is left with

$$\Delta\varphi_S = \omega_\mathrm{C} \oint \frac{\beta g z}{c^2} \mathrm{d}s\,. \tag{3.7}$$

At the dominant order we can approximate $\mathrm{d}s$ by the coordinate time $\mathrm{d}t$ and a short calculation shows that the phase shift is non zero but given by

$$\Delta\varphi_S = \beta\, k\, g\, T^2\,. \tag{3.8}$$

[6] As is clear from e.g. (3.1), the constant f_0 can be absorbed into a rescaling of the coordinates by posing $x^\mu_\mathrm{new} = f_0\, x^\mu_\mathrm{old}$. Henceforth we set this constant to $f_0 = 1$.

Adding the term due to the interaction with lasers which is standard (because WEP is valid) and given by (2.25), we obtain the total phase shift

$$\Delta\varphi = \Delta\varphi_S + \Delta\varphi_\ell = (1+\beta)\, k\, g\, T^2\,, \tag{3.9}$$

where β takes the same meaning as the redshift violation parameter entering (3.5).

Comparing the predictions (3.5) and (3.9) we conclude that within this formalism atom-interferometric gravity measurements would test the gravitational redshift, and would improve existing limits on this measurement by four orders of magnitude, as claimed by MPC.

However, let us now critically examine the implications and meaning of this formalism, and show that it raises a number of difficult fundamental problems. We refer to this formalism (or class of alternative theories) as a multiple Lagrangian framework because for these theories the trajectories of massive bodies (classical particles or classical paths of wave packets) obey a Lagrangian which is different from the Lagrangian used to compute the proper time of clocks or the phase shift of matter waves. Since WEP is valid the motion of massive classical particles obeys the standard Lagrangian of general relativity, $L_{\text{particle}} = L_{\text{GR}} = -mc^2 \mathrm{d}s/\mathrm{d}t$, i.e. to first order

$$L_{\text{particle}} = -m\, c^2 + \frac{GMm}{r_\oplus} - m\, g\, z + \frac{1}{2} m \dot{z}^2\,. \tag{3.10}$$

On the other hand the action integral we have used for the computation of the phase shift of the quantum matter wave is calculated from a different Lagrangian given by

$$L_{\text{wave}} = \left(1 + \beta\, \frac{g\, z}{c^2}\right) L_{\text{particle}}\,. \tag{3.11}$$

We first note that this Lagrangian is not quadratic, and, neglecting higher-order corrections, we recover the Lagrangian used by MPC [34], i.e.

$$L_{\text{wave}} = -m\, c^2 + \frac{GMm}{r_\oplus} - (1+\beta) m\, g\, z + \frac{1}{2} m \dot{z}^2\,, \tag{3.12}$$

where the parameter β that measures the deviation from GR (i.e. $\beta = 0$ in GR) enters as a correction in the atom's gravitational potential energy.

The most important problem is that it is inconsistent to use a different Lagrangian (or metric) for the calculation of the trajectories and for the phases of the atoms or clocks. More precisely, it supposes that the fundamental Feynman path integral formulation of quantum mechanics, which is at the basis of the derivation of the phase shift in an atom interferometer (see Sec. II A), has to be altered in the presence of a gravitational field or could be wrong.[7]

[7] In the case of atom gravimeters using Bloch oscillation [41, 42], also interpreted as redshift tests by MPC, the atoms are "trapped" in optical dipole traps, and the Bloch oscillation frequencies are interpreted as interference of two matter waves at Compton frequency, without using the Feynman formalism. However, this interpretation is plagued with the same difficulties. The Bloch oscillation frequency is obtained from the Schrödinger equation with the Hamiltonian including a gravitational term. We first note that adding or not the rest-mass energy to that Hamiltonian does not modify the derived Bloch frequency, hence the Compton frequency of the atoms plays no role. Second, to obtain the result of MPC one needs to use a different gravitational term in the Hamiltonian when calculating the states and energies providing the Bloch frequency or when calculating the motion of the atoms in the same gravitational field. This is akin to the multiple Lagrangian framework with the same difficulties and inconsistencies.

Physically it amounts to making the distinction between the atoms when calculating their trajectories and the same atoms when calculating their phases, which is inconsistent as it is the same fundamental matter field in both cases. Even more, the basis for writing Eq. (3.6) for the derivation of the atom interferometry phase shifts (as does MPC) is unjustified because the Feynman formalism is violated. To remain coherent some alternative formalism for the atomic phase shift calculations (presumably modifying quantum mechanics) should be developed and used. More generally the multiple Lagrangian formalism supposes that the duality between particles and waves in quantum mechanics gets somehow violated in a gravitational field.

The above violation of the "particle-wave duality", implied by the multiple Lagrangian formalism, is very different from a violation of the equivalence principle in the ordinary sense. In this formalism a single physical object, the atom, is assumed to be described by two different Lagrangians, L_{wave} applying to its phase shift, and L_{particle} applying to its trajectory. By contrast, in tests of the equivalence principle, one looks for the modification of the free fall trajectories or clock rates as a function of composition or clock type. Thus, different bodies or clocks are described by different Lagrangians, but of course for any single type of body we always have $L_{\text{wave}} = L_{\text{particle}}$. The pertinent test-bed for equivalence principle violations is the modified Lagrangian formalism reviewed in Sec. III B, which does not imply any particle-wave duality violation.

The second problem of theories in the multiple Lagrangian formalism and of the interpretation of MPC is the violation of Schiff's conjecture [40]. Indeed, we have assumed in this Section that the LPI aspect of the equivalence principle is violated but that for instance the WEP aspect remains satisfied. In a complete and self-consistent theory of gravitation one expects that the three aspects of the Einstein equivalence principle (WEP, LLI and LPI) are sufficiently entangled together by the mathematical formalism of the theory that it is impossible to violate one without violating all of them. The Schiff conjecture has been proved using general arguments based upon the assumption of energy conservation [38, 39]. A contrario, we expect that violating the conjecture leads to some breakdown of energy conservation. In the present case this translates into postulating two Lagrangians for the same physical object. Explicit theories that are logically and mathematically consistent but still violate Schiff's conjecture are very uncommon (see [15] and references therein).[8]

In conclusion the multiple Lagrangian formalism violates the Feynman path integral formulation of quantum mechanics, the principle of least action and energy conservation, and Schiff's conjecture. Unfortunately, the claim made by MPC [34] that atom gravimeters measure the gravitational redshift can only be substantiated within this formalism.

B. Modified Lagrangian framework

In this Section we review and adapt for our purpose the important "modified Lagrangian framework", which is a powerful formalism for analyzing tests of the various facets of the Einstein equivalence principle (EEP). This formalism allows deviations from GR and metric theories of gravity, with violations of the UFF and UCR, and permits a coherent analysis of

[8] Ni [43] has proposed a possible counterexample of Schiff's conjecture. As it is based on a modified Lagrangian, it falls in the category of theories investigated in Sec. III B, for which atom interferometry does not test the redshift.

atom interferometry experiments. But it does not suffer from the shortcomings of the multiple Lagrangian framework. The formalism describes a large class of non-metric theories, in a way consistent with Schiff's conjecture and fundamental principles of quantum mechanics. Within this broad formalism the atom interferometry experiment is a test of the UFF but not of the redshift or UCR, as we pointed out in our earlier comment [37].

This class of theories is defined by a single Lagrangian, that is however different from the GR Lagrangian (2.23). In particular, the coupling between gravitation and different types of mass-energy is generically not universal [44]. This leads to modifications of the gravitational redshift and also, more generally, to violations of the weak equivalence principle or universality of free fall, the local Lorentz invariance and the local position invariance. But it does not imply a violation of the principle of least action nor of energy conservation, as it is based on a single Lagrangian and, of particular interest here, the Feynman path integral formulation of quantum mechanics remains valid. Most alternative theories commonly considered belong to this class which encompasses a large number of models and frameworks (see e.g. [15, 44] and references therein). It includes for example most non-metric theories (e.g. the Belinfante-Swihart theory [18]), some models motivated by string theory [45] and brane scenarios, some general parameterized frameworks such as the energy conservation formalism [38, 39] (see also [46]), the TH$\varepsilon\mu$ formalism and its variants [47, 48], and the Lorentz violating standard model extension (SME) [49, 50].

The modified Lagrangian formalism is a physical framework in which the violation of EEP originates from the anomalous behaviour of some particular type of energy in a gravitational field. For our purposes it is sufficient to use a strongly simplified "toy" Lagrangian chosen as a particular case within the "energy conservation formalism" of Nordtvedt and Haugan [38, 39] (see [15] for a review). To keep in line with MPC we choose an expression similar to the Lagrangian of GR given by (2.23), namely

$$L_{\text{modified}} = -m(z)\,c^2 + \frac{GMm(z)}{r_\oplus} - m(z)\,g\,z + \frac{1}{2}m(z)\,\dot{z}^2\,. \tag{3.13}$$

We postulate that for the body or atom under consideration the mass m depends on the position z through a violation of the LPI subprinciple of EEP. This is modelled by assuming that a particular internal energy of the atom E_X behaves anomalously in the presence of the gravitational field, where X refers to the type of interaction involved (electromagnetic, nuclear, spin-spin, spin-orbit, etc). In the general case E_X could depend on both the position z and velocity $\dot{z}$ of the atom [39], but here we consider only a dependence on z to model the violation of the LPI. Separating out $E_X(z)$ from the other forms of energies $\overline{E}_Y$ composing the atom and which are supposed to behave normally, we write

$$m(z) = \overline{m} + \frac{1}{c^2}\left[E_X(z) + \sum_{Y\neq X}\overline{E}_Y\right]. \tag{3.14}$$

Here $\overline{m}$ denotes the sum of the rest masses of the particles constituting the atom, and $\overline{m}$ and all $\overline{E}_Y$'s are constant. The violation of LPI is modelled in the simplest way by assuming that at the leading order

$$E_X(z) = \overline{E}_X + \beta_X^{(a)}\,\overline{m}\,g\,z\,, \tag{3.15}$$

where $\beta_X^{(a)}$ denotes a dimensionless parameter characterizing the violation of LPI and depending on the particular type of mass-energy or interaction under consideration, e.g. $\beta_X^{(a)}$

would be different for the electromagnetic or the nuclear interactions, with possible variations as a function of spin or the other internal properties of the atom, here labelled by the superscript (a). Thus $\beta_X^{(a)}$ would depend not only on the type of internal energy X but also on the type of atom (a). Defining now the "normal" contribution to the total mass,

$$m_0 = \overline{m} + \sum_Y \frac{\overline{E}_Y}{c^2}\,, \tag{3.16}$$

and replacing $m(z)$ by its explicit expression into the Lagrangian (3.13) we obtain

$$L_{\text{modified}} = -m_0\,c^2 + \frac{GMm_0}{r_\oplus} - \left(1+\beta_X^{(a)}\right) m_0\,g\,z + \frac{1}{2}m_0\,\dot{z}^2\,, \tag{3.17}$$

in which we have neglected some higher-order relativistic terms. A major difference with the Lagrangian (3.12) of the previous Section is that the coefficient $\beta_X^{(a)}$ is not universal but depends on the internal structure of the atom.

Before discussing the atom interferometry experiment, let us recall the results for traditional free fall and redshift experiments. By varying (3.17) we obtain the equations of motion of the atom as

$$\ddot{z} = -\left(1+\beta_X^{(a)}\right) g\,, \tag{3.18}$$

which shows that the trajectory of the atom is affected by the violation of LPI and is not universal. In fact we see that $\beta_X^{(a)}$ measures the non-universality of the ratio between the atom's passive gravitational mass and inertial mass. Thus, in the modified Lagrangian framework the violation of LPI implies a violation of WEP and the UFF, and $\beta_X^{(a)}$ appears to be the UFF-violating parameter. This is a classic example [38, 39] of the validity of Schiff's conjecture, namely that it is impossible to violate LPI (or LLI) without also violating WEP.

The violation of LPI is best reflected in classical redshift experiments with clocks which can be analysed using a cyclic gedanken experiment based on energy conservation [15, 38, 39]. The result for the frequency shift in a Pound-Rebka type experiment is

$$Z = \left(1+\alpha_X^{(a)}\right) \frac{g\,\Delta z}{c^2}\,, \tag{3.19}$$

where the redshift violating (or UCR violating) parameter $\alpha_X^{(a)}$ is again non-universal. As such it has to be carefully distinguished from the redshift violation parameter β appearing in Eq. (3.5) of the previous Section [see also the discussion after (3.5)]. The important point, proven in Refs. [15, 38, 39], is that within the framework of the modified Lagrangian (3.17), the UCR violating parameter $\alpha_X^{(a)}$ is related in a precise way to the UFF-violating parameter $\beta_X^{(a)}$, namely

$$\beta_X^{(a)} = \alpha_X^{(a)}\,\frac{\overline{E}_X}{\overline{m}\,c^2}\,. \tag{3.20}$$

Therefore tests of UCR and UFF are not independent, and we can compare their different qualitative meaning. Since for typical energies involved we shall have $\overline{E}_X \ll \overline{m}\,c^2$ this means that $\beta_X \ll \alpha_X$.[9] For a given set of UFF and UCR tests their relative merit is given

[9] We denote by β_X and α_X some typical values of the parameters $\beta_X^{(a)}$ and $\alpha_X^{(a)}$ for different bodies (a).

by Eq. (3.20) and is dependent on the model used, i.e. the type of anomalous energy E_X and its dependence on the used materials or atoms.

For example, let us assume a model in which all types of electromagnetic energy are coupled in a non-universal way, i.e. $\beta_{\rm EM} \neq 0$ (with all other forms of energies behaving normally), and where the clock transition is purely electromagnetic. The UFF test between two materials (a) and (b), both containing electromagnetic energy, is carried out with an uncertainty of $|\beta_{\rm EM}^{(a)} - \beta_{\rm EM}^{(b)}| \simeq |\beta_{\rm EM}| \lesssim 10^{-13}$ [6, 7]. On the other hand the UCR test for a clock of type (a) based on an electromagnetic transition, is carried out with an uncertainty of $|\alpha_{\rm EM}^{(a)}| \simeq |\alpha_{\rm EM}| \lesssim 10^{-4}$ [11]. For macroscopic test bodies the nuclear electromagnetic binding energy contributes typically to about $\overline{E}_{\rm EM}/(\overline{m}c^2) \simeq 10^{-3}$ of the total mass [38], so we may have $|\beta_{\rm EM}| \simeq 10^{-3}\,|\alpha_{\rm EM}|$, which yields for the UFF test a limit of about $|\alpha_{\rm EM}| \lesssim 10^{-10}$. This is still a much more stringent limit than the UCR test $|\alpha_{\rm EM}| \lesssim 10^{-4}$. So in such a model UFF tests are significantly more sensitive than UCR tests.

However, that result depends on the particular model used. If we assume another model in which the nuclear spin plays a role leading to a non-universal coupling of atomic hyperfine energies, i.e. $\beta_{\rm HF} \neq 0$ (with other forms of energies and properties of the atom behaving normally), the result is different. Atomic hyperfine energies are of order 10^{-24} J (corresponding to GHz transition frequencies) which for typical atomic masses leads to $\bar{E}_{\rm HF}/(\bar{m}c^2) \simeq 10^{-16}$. As a consequence UFF tests set a limit of only $|\alpha_{\rm HF}| \lesssim 10^{3}$ while UCR tests using hyperfine transitions (e.g. H-masers [11]) set a limit of about $|\alpha_{\rm HF}| \lesssim 10^{-4}$. The conclusion is therefore radically different in this model where UCR tests perform orders of magnitude better than UFF tests.

To summarize, the two types of tests, UFF and UCR, are complementary in the modified Lagrangian framework, and need to be pursued with equal vigor, because depending on the model used either one of the tests can prove significantly more sensitive than the other. A corollary is that usual tests of UCR do not need a violation of Schiff's conjecture to be meaningfully interpreted. They can perfectly be contrasted with UFF tests within a formalism satisfying the Schiff conjecture. This is contrary to the "redshift test" proposed by MPC which is meaningful only in hypothetical frameworks violating the Schiff conjecture such as the multiple Lagrangian formalism of Sec. III A.

Indeed, let us now consider the application to atom interferometry. In the experiment of Refs. [29, 34] the "atom-clock" that accumulates a phase is of identical composition to the falling object (the same atom), hence one has to consistently use the same value of $\beta_X^{(a)}$ when calculating the trajectories and the phase difference using the Lagrangian (3.17) inserted into (2.14). It is then easy to show that $\Delta\varphi_S = 0$ with the above Lagrangian [21–23]. The vanishing of $\Delta\varphi_S$ in this case is a general property of all quadratic Lagrangians and comes from consistently using the same Lagrangian for the calculation of the trajectories and the phase shift, as shown in Sec. II B above. It is related to the cancellation between the special relativistic effect [last term in (3.17)] and the gravitational potential energy [third term in (3.17)], discussed in more detail below.

Then the total phase shift of the atom interferometer is again given by the light interactions only, which are obtained from the phases at the interaction points evaluated using the trajectory given by (3.17). From Eq. (2.25) one then obtains

$$\Delta\varphi = \Delta\varphi_\ell = \big(1 + \beta_X^{(a)}\big) k\, g\, T^2\,. \tag{3.21}$$

We first note that in this class of theories the Compton frequency plays no role, as $\Delta\varphi_S = 0$. Second, we note that although $\beta_X^{(a)}$ appears in the final phase shift, this is entirely related to the light phase shift coming from the trajectory of the atoms, and thus is a measurement of the effective free fall acceleration $(1+\beta_X^{(a)})g$ of the atoms, which is given by Eq. (3.18). In Refs. [29, 34] the resulting phase shift is compared to $k\,\tilde{g}\,T^2$ where $\tilde{g}$ is the measured free fall acceleration of a falling macroscopic corner cube, i.e. $\tilde{g} = \left[1+\beta_X^{(\text{corner cube})}\right]g$, also deduced from Eq. (3.18). In this class of theories the experiment is thus a test of the universality of free fall, as it measures the differential gravitational acceleration of two test masses (Cæsium atom and corner cube) of different internal composition, with precision

$$\left|\beta_X^{(\text{Cs})} - \beta_X^{(\text{corner cube})}\right| \lesssim 7\times 10^{-9}\,, \tag{3.22}$$

for any kind of internal energy X. But it is *not* a test of the gravitational redshift or universality of clock rates. As already mentioned the vanishing of $\Delta\varphi_S$ and thus the non-sensitivity to the gravitational redshift in the "atom-clock" phase is related to the cancellation between the special relativistic term and the gravitational term when integrating along the trajectories. More generally, the underlying fundamental point here is that for a measurement of the gravitational redshift one wants to determine the non-zero effect of the gravitational redshift term [third term in (2.23) or (3.17)] on the clock phase (atom or classical) *independently* of the effect of the special relativistic term [fourth term in (2.23) or (3.17)] and of possible other phase shifts (e.g. the light phase $\Delta\varphi_\ell$). This is generally done by using independent measurements that provide the knowledge of the trajectories of the clocks $z(t)$ and their velocities $\dot{z}(t)$. One uses that knowledge to subtract the time dilation and light phase shifts from the overall measured shift $\Delta\varphi$. The remaining gravitational redshift term can then be compared to the GR prediction.

As an example, in the Pound-Rebka experiment [8, 9] the two clocks were fixed to the Earth's surface at the top and bottom of the tower, which imposes $z(t) = \text{const}$ and $\dot{z}(t) = 0$. As a result the first-order Doppler shift was zero (implying $\Delta\varphi_\ell = 0$) and the special relativistic term in the action phase shift also.[10] The measured frequency or phase difference was then entirely due to the gravitational term and could be compared to the GR prediction, clearly favoring GR. In modern rocket or satellite experiments [11, 12] the trajectories of the clocks are measured independently using radio and/or laser ranging which allows calculation and correction of the special relativistic and light phase-shifts, and thus an unambiguous measurement of the gravitational redshift. In contrast, in the atom-interferometry experiment the trajectories are not measured independently but theoretically derived from the Lagrangian and initial conditions, thus making a "pure" measurement of the gravitational term impossible. One might even argue that in such an experiment the independent determination of the wave packet trajectories (e.g. by high resolution imagery) would be impossible as it would destroy the interference at the interferometer exit according to one of the fundamental tenets of quantum mechanics.

IV. CONCLUSION

We have shown that it was possible to provide answers to the question asked in the title of this article provided that the alternative theory or theoretical framework is carefully

[10] Actually, in the real experiment the measured signal is given by a first-order Doppler shift which is applied to compensate for the gravitational redshift.

specified. To this aim, after having proved in Sec. II that within GR atom interferometers are insensitive to the gravitational redshift, we have considered two classes of alternative theories to GR.

The first class of models is based on a modified Lagrangian (see Sec. III B). In this class one finds most alternative approaches that postulate a non-universal coupling between gravity and other fields of the standard model of particle physics or its extensions (see e.g. [44, 47–50]), including some versions of string theory, brane scenarios, parameterized frameworks, etc. This class of models is consistent with Schiff's conjecture and represents the most natural framework for analyzing and comparing the various consequences of the violation of the equivalence principle. We have shown that in this class the atom interferometry experiment is not a test of the gravitational redshift, or more generally of the universality of clock rates. Instead it is a test of the weak equivalence principle or universality of free fall at the 7×10^{-9} accuracy level, see Eq. (3.22). As such it is less accurate than other UFF tests [6, 7] but still interesting as it is the best test of this kind which compares the free fall of quantum objects to that of classical test masses.

The second class of models relies on multiple Lagrangians (Sec. III A). Within this class, the atom interferometry experiment can be seen as a test of the gravitational redshift, in line with the claims of MPC [34], but at the expense of extreme conceptual difficulties: the basic principles of quantum mechanics, such as the Feynman path integral formulation and Schrödinger's equation, are affected, as well as energy conservation and the Schiff conjecture. This is because two Lagrangians are used in this class of models, one for the trajectory of the atoms and another one for the phase of the associated matter field. In addition, these models pose a fundamental problem for calculating phases in atom interferometry tests, since the standard formula for phase shifts is based on principles of quantum mechanics which are violated in this class of alternative theories.

We conclude that the general statement of MPC according to which the atom interferometer measures or tests the gravitational redshift is incorrect. Instead, the interpretation of the experiment needs to be considered in the light of alternative theories or frameworks. Although one can consider particular alternative frameworks in which the statement of MPC could make sense, such frameworks raise unacceptable conceptual problems which are not at the moment treated in a satisfactory manner. In particular, they break the fundamental principles of quantum mechanics which they use for calculating matter wave phases. In most common and plausible theoretical frameworks, the atom interferometry experiment does not test the redshift or universality of clock rates, but only tests the universality of free fall.

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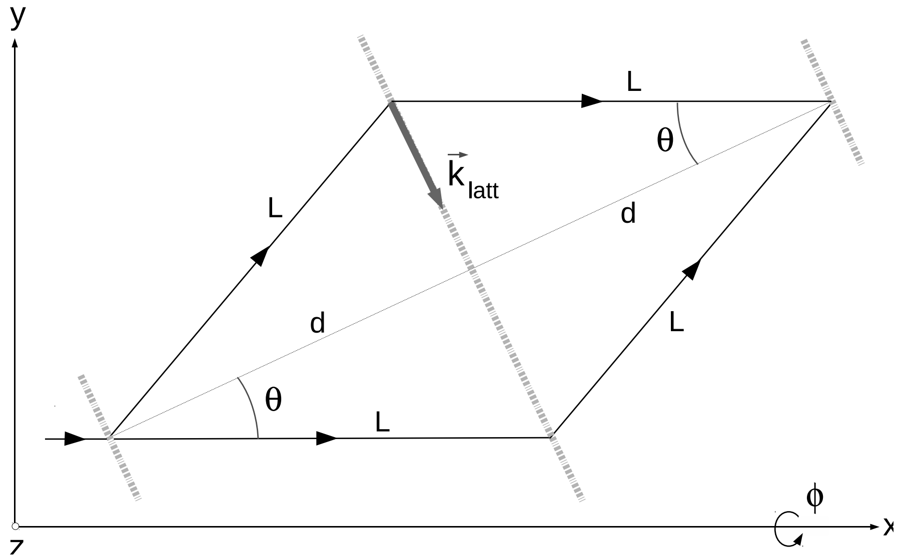

FIG. 2: Geometry of the neutron interferometer, simplified from Fig. 1 of Ref. [27]. Thick dotted lines represent the scattering crystal planes, thin solid lines the neutron trajectories.

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APPENDIX A: PHASE SHIFT IN NEUTRON INTERFEROMETRY

In this Appendix we show that the phase shift induced by gravity in neutron interferometers is given by an expression similar to that in atom interferometry. In consistency with the analogy between neutron and atom interferometry pointed out in Ref. [51], the gravitational phase shift is independent of the Compton frequency also for the neutrons.

Neutron interferometers have been first used for the measurement of the gravitational acceleration in 1975 [27]. A neutron beam is Bragg scattered by three silicon crystal planes and forms an interferometer as shown in Fig. 2. The phase shift at the exit of the interferometer is given in Ref. [27] to leading order as

$$\Delta\varphi = \frac{\lambda_{\rm dB} M^2\, d^2\, g}{\pi\hbar^2} \tan\theta\, \sin\phi\,, \tag{A1}$$

where $\lambda_{\rm dB}$ and M are the neutron de Broglie wavelength and mass respectively. Using the definition of the de Broglie wavelength $2\pi\hbar/\lambda_{\rm dB} = Mv$, where v is the velocity of the neutrons, Eq. (A1) can be rearranged to

$$\Delta\varphi = \frac{4\pi g}{\lambda_{\rm dB}} T^2 \sin\theta \cos\theta \, \sin\phi \,, \tag{A2}$$

with $T \equiv L/v$. The well known Bragg condition for the scattering at the crystal planes is

$$\lambda_{\rm dB} = 2a \, \sin\theta \,, \tag{A3}$$

with a the atomic spacing in the crystal lattice. Substituting Eq. (A3) into Eq. (A2) one easily obtains

$$\Delta\varphi = k_{\rm latt} \, g \, T^2 \, \cos\theta \sin\phi = \boldsymbol{k}_{\rm latt} \cdot \boldsymbol{g} \, T^2 \,, \tag{A4}$$

where $k_{\rm latt} \, \cos\theta \sin\phi = (2\pi/a) \, \cos\theta \sin\phi$ is the projection of the lattice wave vector $\boldsymbol{k}_{\rm latt}$ onto the direction of the gravitational field $\boldsymbol{g}$.

We note that Eq. (A4) is formally identical to Eq. (2.26) with the lattice wave vector playing the role of the laser wave vector. In both cases, the expression for the phase shift is independent of the mass of the atoms (neutrons) or the associated Compton frequency. In both cases, the interferometer is to be understood as measuring the free fall of the atoms (neutrons) using the laser (crystal lattice) as a "ruler".

APPENDIX B: EFFECT OF HIGH-ORDER GRAVITY GRADIENTS

As we have seen, for any quadratic Lagrangian the difference of classical actions (2.14) is zero or reduces to the contribution of internal energies. In this Appendix we estimate the magnitude of the effect of including cubic terms in the Lagrangian of GR, due to second-order gravity gradients, i.e.

$$L_{\rm GR}(z, \dot{z}) = -mc^2 + \frac{GMm}{r_\oplus} + mg\Big[-z + \frac{z^2}{r_\oplus} - \frac{z^3}{r_\oplus^2}\Big] + \frac{1}{2} m \dot{z}^2 \,. \tag{B1}$$

For such cubic Lagrangian we expect that the difference of classical actions will no longer be zero. On dimensional grounds we expect that the result should then be given by[11]

$$\Delta\varphi_S = \frac{\Delta S_{\rm cl}}{\hbar} = C \, k \, g \, T^2 \left(\frac{\hbar \, k \, T}{m \, r_\oplus}\right)^2 + \Delta\varphi_{gg'} \,, \tag{B2}$$

where C denotes some global coefficient of the order of one. By integrating the Lagrangian (B1) along the classical paths one obtains $C = 1/4$. We note that for the Lagrangian (B1) the interferometer closes up only when we adjust the time T' to a value that is different from T. So the interferometer is no longer symmetric and there is a contribution coming from the difference of internal energies gg' of the atoms, given by

$$\Delta\varphi_{gg'} = \omega_{gg'} \frac{gT^3}{r_\oplus} \,. \tag{B3}$$

The first term in (B2) gives a non zero contribution which could be interpreted as a redshift effect by analogy with classical clock experiments. However, we insist that in order

[11] To (B2) we should also add the contribution of $\Delta\varphi_\ell$.

to compute correctly the phase shift one would have to revisit the derivation made in Sec. II A for the case of cubic and higher-order Lagrangians. If we assume that at least the order of magnitude given by the first term in (B2) is correct, the effect is extremely small. With $\hbar k/m \simeq 3\,\mathrm{mm/s}$, $T \simeq 0.1\,\mathrm{s}$ and $r_\oplus = 6400\,\mathrm{km}$, it gives a relative correction to the main effect (i.e. $\Delta\varphi_\ell = kgT^2$) of the order of 2×10^{-21}, much too small to be measured in the atom interferometry experiments [29, 30, 34] whose current precision is about 7×10^{-9}. Therefore we conclude that the redshift effect in atom interferometry, if it exists at all, is very small and currently undetectable.